Balancing Sustainability, Profitability, and Resiliency in a 2-Prey, 1-Predator Fishery

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Background and Purpose

Species of fish in fisheries, areas where fish are harvested for commercial purposes, are often part of a complex ecological community. In these marine communities, species interact primarily through competition and predation. Consider a system made of three species: two prey and one predator. It is assumed that the predator preys on both species of prey within the system. Further, it is assumed that the two prey are in competition. This project investigates three harvesting management approaches on a theoretical two-prey one-predator fishery system, namely, the maximum sustainable yield (MSY), the maximum economic yield (MEY), and the resilience-maximizing yield (RMY). While all three are theoretical top catch or yield that can be harvested in the long-term on a particular species, they differ in perspectives depending on the underlying goal. MSY considers harvesting for yield so that species are not driven to extinction (an environment sustainability goal), MEY considers harvesting for yield to guarantee profitability (an economic goal), while RMY considers the resilience of a system, which is the ability of a system to sustain perturbations of an equilibrium state. This project aims to address the MSY, MEY, and RMY management issues on all eight possible harvesting scenarios in a 2-prey, 1-predator fishery.

Mathematical Model

Suppose $x_1(t)$, $x_2(t)$, $x_3(t)$ represent population counts of two competing prey species and a predator species, respectively.

$$\begin{cases} \dfrac{dx_1}{dt} = \lambda_1 x_1\left(1-\dfrac{x_1}{K_1}\right) - \alpha_{12}x_1x_2 - \alpha_{13}x_1x_3 - q_1E_1x_1 \\ \dfrac{dx_2}{dt} = \lambda_2 x_2\left(1-\dfrac{x_2}{K_2}\right) - \alpha_{21}x_1x_2 - \alpha_{23}x_2x_3 - q_2E_2x_2 \\ \dfrac{dx_3}{dt} = \alpha_{31}x_1x_3 + \alpha_{32}x_2x_3 - x_3^2 - q_3E_3x_3 \end{cases}$$

λ_1	growth parameter of x_1
λ_2	growth parameter of x_2
K_1	environmental carrying capacity of x_1
K_2	environmental carrying capacity of x_2
α_{12}	competition effect of x_2 on x_1
α_{21}	competition effect of x_1 on x_2
α_{13}	predation effect of x_3 on x_1
α_{23}	predation effect of x_3 on x_2
α_{31}	predation effect on x_3 by preying on x_1
α_{32}	predation effect on x_3 by preying on x_2

c_1	fishing cost per unit effort for x_1
c_2	fishing cost per unit effort for x_2
c_3	fishing cost per unit effort for x_3
p_1	price per unit biomass of x_1
p_2	price per unit biomass of x_2
p_3	price per unit biomass of x_3
q_1	catchability coefficient of x_1
q_2	catchability coefficient of x_2
q_3	catchability coefficient of x_3
E_1	harvesting effort on x_1
E_2	harvesting effort on x_2
E_3	harvesting effort on x_3

Harvesting Analysis

With three species in the system, 8 harvesting scenarios:

	E_1	E_2	E_3	Harvesting Scenario
Case 1	$= 0$	$\neq 0$	$\neq 0$	Prey 1 is not harvested.
Case 2	$= 0$	$= 0$	$\neq 0$	Prey 1 and prey 2 are not harvested.
Case 3	$\neq 0$	$= 0$	$\neq 0$	Prey 2 is not harvested.
Case 4	$\neq 0$	$\neq 0$	$\neq 0$	All species are harvested.
Case 5	$= 0$	$= 0$	$= 0$	No harvesting.
Case 6	$\neq 0$	$\neq 0$	$= 0$	Predator is not harvested.
Case 7	$= 0$	$\neq 0$	$= 0$	Prey 1 and predator not harvested.
Case 8	$\neq 0$	$= 0$	$= 0$	Prey 2 and predator not harvested.

Harvesting Goals

Suppose $(x_1(E), x_2(E), x_3(E))$ is the interior coexistence equilibrium.

$$x_1 = \frac{M_1}{M}, \quad x_2 = \frac{M_2}{M}, \quad x_3 = \frac{M_3}{M},$$

$$M = \begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \frac{\lambda_2}{K_2} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & -1 \end{vmatrix}$$

$$M_1 = \begin{vmatrix} \lambda_1 - q_1E_1 & \alpha_{12} & \alpha_{13} \\ \lambda_2 - q_2E_2 & \frac{\lambda_2}{K_2} & \alpha_{23} \\ q_3E_3 & \alpha_{32} & -1 \end{vmatrix}$$

$$M_2 = \begin{vmatrix} \frac{\lambda_1}{K_1} & \lambda_1 - q_1E_1 & \alpha_{13} \\ \alpha_{21} & \lambda_2 - q_2E_2 & \alpha_{23} \\ \alpha_{31} & q_3E_3 & -1 \end{vmatrix}$$

$$M_3 = \begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{12} & \lambda_1 - q_1E_1 \\ \alpha_{21} & \frac{\lambda_2}{K_2} & \lambda_2 - q_2E_2 \\ \alpha_{31} & \alpha_{32} & q_3E_3 \end{vmatrix}$$

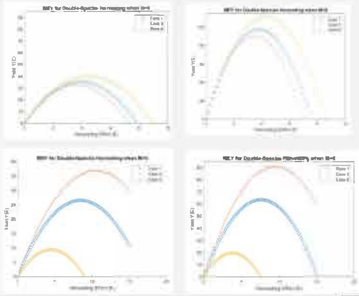
MSY: optimize yield as a function of the harvesting effort E, given by $Y(E) = (x_1(E) + x_2(E) + x_3(E))E$.
MEY: optimize economic rent as a function of E,
$P(E) = (P_1(E)+P_2(E)+P_3(E))E$, where $P_i(E) = (p_1q_1x_1(E)-c_i)$.
RMY: find the leading eigenvalue of the Jacobian of the system at a stable coexistence equilibrium

Numerical Simulations

Assumed parameter values:

Parameters	Values for $M > 0$	Values for $M < 0$
$\alpha_{12} = \alpha_{21}$	1.9	0.01
$\alpha_{13} = \alpha_{23}$	0.7	0.1
$\alpha_{31} = \alpha_{32}$	1.1	0.5
(λ_1, λ_2)	(8.5, 9.5)	(8.5, 9.5)
$K_1 = K_2$	10	10
E	1	1
$p_1 = p_2 = p_3$	3	3
$q_1 = q_2 = q_3$	1	1
$c_1 = c_2 = c_3$	1	1

MSY, MEY on double-species harvesting:



MSY, MEY on single-species harvesting:

Single-species $M > 0$	$(E^*, \text{MSY}(E^*))$	$(\bar{E}, \text{MEY}(\bar{E}))$
Case 2 $(E_1, E_2, E_3) = (0, 0, 1)$	(3.5424, 8.0983)	(3.2841, 20.8817)

Single-species $M < 0$	$(E^*, \text{MSY}(E^*))$	$(\bar{E}, \text{MEY}(\bar{E}))$
Case 2 $(E_1, E_2, E_3) = (0, 0, 1)$	(4.9449, 22.0242)	(4.7598, 61.2201)
Case 7 $(E_1, E_2, E_3) = (0, 1, 0)$	(4.4667, 20.0312)	(4.3007, 55.7100)
Case 8 $(E_1, E_2, E_3) = (1, 0, 0)$	(3.9650, 17.5382)	(3.8156, 48.7242)

MSY, MEY on combined harvesting:

All-species $M < 0$	$(E^*, \text{MSY}(E^*))$	$(\bar{E}, \text{MEY}(\bar{E}))$
Case 4 $(E_1, E_2, E_3) = (1, 1, 1)$	(3.6302, 48.5060)	(3.4944, 134.8312)
All-species $M > 0$	**$(E^*, \text{MSY}(E^*))$**	**$(\bar{E}, \text{MEY}(\bar{E}))$**
Case 4 $(E_1, E_2, E_3) = (1, 1, 1)$	(3.3829, 14.7642)	(2.9953, 34.7255)

RMY for $M < 0$: t is he negative reciprocal of the leading eigenvalue.

Harvesting Case	Coexistence Equilibrium	Largest Negative Eigenvalue	t
$(0, E_2, E_3)$	(9.0183, 8.0589, 7.5386)	-7.5754	0.1320
$(0, 0, E_3)$	(8.9514, 9.0629, 8.0071)	-8.0164	0.1247
$(E_1, 0, E_3)$	(7.8358, 9.1299, 7.4828)	-7.5101	0.1332
(E_1, E_2, E_3)	(7.9027, 8.1258, 7.0143)	-7.1368	0.1401
(0, 0, 0)	(8.8465, 8.9692, 8.9079)	-7.8966	0.1266
$(E_1, E_2, 0)$	(7.7979, 8.0321, 7.9150)	-7.0077	0.1427
$(0, E_2, 0)$	(8.9134, 7.9652, 8.4393)	-7.4873	0.1336
$(E_1, 0, 0)$	(7.7309, 9.0361, 8.3835)	-7.2695	0.1376

Conclusion

- Management decisions on sustainable harvesting of any species in our marine ecosystems benefit from modeling and simulations due to the underlying complex ecological interactions between species.
- Harvesting effort required to realize MEY is less than the required to realize MSY: increasing effort beyond MEY-effort may not provide more economic benefits and may cause sustainability issues.
- Single-species harvesting, M < 0: predator-oriented harvesting presents highest MEY-level but prey-oriented harvesting is most resilient.
- Double-species harvesting, M < 0: prey-oriented harvesting presents highest MEY-level but predator-oriented harvesting is most resilient.
- There is a tradeoff between yield and resilience.

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