EDITOR'S NOTES

Greetings from West Point!

This issue explores the complex issues regarding evaluation and assessment at the Service Academies. It has been most interesting and informative to read the submissions and has served as a learning tool more than an additional duty. As teachers, we have much to learn from one another. Taking the time to read this issue is a move toward that goal.

The ideas and opinions expressed in these articles are not just words on paper but are actively and carefully implemented. In addition, if a concept does not work well, it is extensively re-evaluated and adjusted or discarded, as the situation warrants. The dedication of the authors is laudable.

Those of us who have obtained our education and have taught other than at a Service Academy have come to appreciate this unique experience. Remember the good ol' days being stuffed into a lecture hall with 150 other students? Recall the "No, I don't have office hours now…?" Remember the evening classes, with their three-hour lecture periods? Well, not at the Service Academies. The Academies are dedicated to the students and to their education. Read on for proof…

Best wishes from West Point!
Mary Jane Graham

CONTENTS

SUBSCRIPTIONS TO *MATHEMATICA MILITARIS*:

If you would like to be on our mailing list, please send your name, address, and affiliation to:

Dr. Mary Jane Graham or MAJ Phillip Beaver

Department of Mathematical Sciences

United States Military Academy
ATTN: MADN-A
West Point, New York 10996

Assessment in Little Bites

MAJ Marie Revak
USAFA Center for Educational Excellence

When you think of student assessment, do you think of exams, quizzes, and graded projects? Why not literally take a "little bite" of class time and do an informal, formative assessment? The purpose of formative assessment is to improve the quality of student learning, not to assign a grade. Formative assessments can provide instructors with valuable information on what, how much, and how well students are learning.

One type of formative assessment is the "minute paper," so named because the assessment can be accomplished in about a minute (or two or three). Minute papers are flexible enough to fit any class at any time. Here are some examples of questions you can ask with minute papers. Feel free to adapt these questions and share them with others.

1. What was the muddiest point in today's lesson (or lecture)?
2. What was the most important thing (or 2, 3, 4, or 5 most important things) you learned in today's lesson?
3. What is one important question that you'd like to pursue further?
4. What question is uppermost in your mind at the end of today's lesson?
5. What important question did the problems (or assignment, or project) make you ask of yourself?
6. What was the most surprising information about ________?
7. Give an example of ________.
8. Give an application of ________.
9. Explain ________ in your own words.

You can also use minute papers to obtain feedback on your teaching. A simple "start, stop, continue" minute paper, in which students are asked to write one thing you should start doing, one thing you should stop doing, and one thing you should continue doing, provides valuable information. Or you can simply ask students which classroom activities provide the best learning opportunities.

Be sure to take time to discuss the feedback provided by your students. You'll get better feedback if your students know that you take their responses seriously.

Assessing Mathematical Learning Without Grading

LTC Dick Jardine, USMA

For most teachers, grades first come to mind with the mention of educational assessment. But there is great value in ungraded assessment in the mathematics classroom. Ungraded assessments provide invaluable feedback for both the teacher and the learner, feedback which enhances the learning process. Some ungraded assessment methods I have used, or learned about in recent years, are described below.

One activity that works for me is the Minute Paper. The idea of a Minute Paper is to have students answer the following two questions during the last minute of class:

1. What was the most useful or meaningful thing you learned during this class?

2. What question remains unanswered as we end this class?

Provide each student the questions on a handout at the beginning of the class hour. The idea is that knowing they have to answer the questions at the end of class will increase student attention level during class. They are informed that the exercise is anonymous, reducing student stress. (After doing the activity enough times, it is easy just to have the students write the answers on a sheet of notebook paper). I have found the feedback very enlightening, as the student responses always give me better insight to what they feel confident about and what troubles them. (Well, almost always. One student's response to the second question was, "What kind of stone is in your class ring?")

Assessment is meaningless unless the results are used to improve the learning. In reviewing the Minute Papers after class, it is not hard to identify common concepts about which students remain uncomfortable. Feedback should be given to the students as soon as possible. Examples of appropriate feedback include identifying the specific section of the text students should review and recommending exercises they should work to build student confidence in their understanding of the topic. Email works well for that purpose. Additionally, the student responses may indicate the appropriate topic for a brief lecture at the start of the next class meeting. Remember that we teach people, not mathematics, and it is important that we respond to our students' needs and help them learn concepts that are within their grasp.

At USMA, sending students to the boards is the best of the ungraded assessment activities. Our classrooms are paneled with chalk- or white-boards on every wall, enough for each student to have room to work at a board. While students work at the boards, the instructor can assess student difficulties both collectively and individually. If just one student, or one student group, has a difficulty, the instructor can devote attention to that student or group. If a significant number of students have a common difficulty with a particular concept, then that is the instructor's cue for a mini-lecture on that topic. Boardwork is active learning at its best, with students engaged in doing mathematics and instructors coaching based on on-the-spot assessment of student performance. (A significant "plus" to boardwork is that it is difficult for students to sleep while working at the boards, although some try.) If there is insufficient board space for all students in your classroom, similar assessment can be achieved with students working in groups at tables, while the instructor flits from group to group, assessing student performance and helping students solve difficulties in the process.

When preparing assessment activities, the key thought should be, "How will doing this activity increase my students' learning?" Coaching students at the boards or while they are seated in small groups is an easy way to give students confidence that they are learning mathematics because they are doing the work. Reviewing students' ungraded homework is another way to see whether they are doing the work. Prior to students beginning their work at the boards, or work at their desks, have them place their homework exercises on a corner of their desk. Then check the work. It doesn't take much more than a quick glance to assess who is getting the work done and who is not, and it is appropriate to give immediate feedback to the students.

There are other ungraded assessment activities that can enhance learning in the mathematics classroom. Concept maps and learning logs are just two that I have learned while attending seminars sponsored by the Center for Teaching Excellence at West Point. There are also many relevant ideas in:

Angelo, T. A. & Cross, K.P. Classroom Assessment Techniques: A Handbook for College Teachers, 2nd edition. San Francisco: Jossey-Bass, 1993, pp. 183-187.

Try to adapt ideas that work best for you and your students. For example, if the Minute Paper described above doesn't work for you because it takes too much time, adapt it by asking a shorter question, such as "What point was the least clear to you in today's lesson?" Try not to think only about grading when it comes to assessment in the mathematics classroom. Some of the best assessment methods, those that contribute the most to making us effective teachers and our students effective learners, are not graded.

Encouraging Self-Assessment

David J. Haroldsen, Ph.D., USMA

"My job is to make myself unnecessary." This statement by my undergraduate analysis teacher reflects the idea that an education is not so much a set of information as it is a collection of tools for examining and analyzing the world. When does the instructor become unnecessary? In part, when the student is able to assess his or her level of understanding and proficiency. Without question, the best students academically are almost always those who are best at analyzing their understanding and weaknesses in the subject matter. One of our tasks as instructors should be to teach our students how to assess themselves in all areas of their conduct - academic, professional, and personal. Teaching a student self-assessment should be integrally linked with the overall assessment process. Indeed, it is possible to view much of our work as instructors as teaching our students how to know when they understand and (more importantly) how to tell when they don't understand.

Perhaps nothing typifies the challenge inherent in teaching students to assess themselves than a conversation with a cadet after our first calculus midterm examination. One question on the test which required a good conceptual understanding of the derivative proved vexing to most of the cadets. After the test, I reviewed his score and discussed the correct solution of the problem with him. He challenged the validity of the question by asserting "I think I understand the concept of the derivative quite well, and I don't see any relationship between this problem and the derivative." My response: "You evidently don't understand the derivative as well as you think you do!"

Following are four questions which I regularly posed to students in my first-year calculus course to encourage them to assess their own understanding and progress.

1. *Did you check your answer?*

While this may seem obvious, I was surprised to find how difficult it was to impress upon my first year calculus students the need to check their results in as many ways as possible. I have found this to be a useful self-assessment tool for several reasons. First, the student gets immediate feedback about the results of their method, even if they can't ask the instructor or check the answer in the back of a textbook. On a procedural level, they can readily deduce whether they have done the problem correctly. I have also found that by appropriately asking this question, I can help a student gauge their understanding of a particular concept. Many students did not have a good grasp of the meaning of a differential equation. Even though they could "turn the crank" and generate a solution, they had difficulty relating the solution to the equation at a conceptual level. By encouraging them to verify their solutions, they gained greater insight into the concept of a differential equation and what it means to "solve" the equation.

2. *Does it make sense?*

It is not always possible to analytically verify answers to problems. Moreover, it is easy to make a mistake in the verification process, particularly if you are confident that you know what you are doing. Hence, a necessary corollary to the verification is to critically examine the answer. This question is particularly useful in the analysis of applications and real-world problems where units play an important role. On a quiz near Thanksgiving Day, I used a problem involving Newton's law of cooling to test several concepts. The idea was simple: take a frozen turkey out of the freezer (20 degrees) and let it defrost at room temperature (68 degrees). What will the temperature be a few hours later? When I graded the quizzes, I was dismayed to find a significant number of students whose answered outside the realm of reason. (The answers ranged from –40 degrees to 20 million degrees.) My students were so consumed in calculating a number that they failed to examine the significance of the number they computed. When we reviewed the problem in class, I told them to ignore all of the equations and tell me what a reasonable answer would be. Most could immediately identify an appropriate range of temperatures, but had not even thought about it during the quiz. On graded work, I have learned to encourage and reward statements such as "I can't figure out the correct answer, but I know this is wrong because…" This is done in order to encourage students to think critically about their work.

3. *Do you really believe your answer? Why?*

Both in and out of the classroom, I often use this question to encourage a student to reflect on their work and results. It is often tempting to watch a student struggle through a calculation and then simply tell them that their work is correct or incorrect. Asking the student whether they really believe their answer, even if their work *is* correct, forces the student to review their work and gives them an opportunity to reflect on the concepts involved. The follow up question (Why?) requires them to articulate their reasoning and justify their methods. This is especially important for those students struggling with knowing which methods to use on a given problem. These students tend to grab the first mathematical tool which comes to mind and vigorously launch themselves into a misguided attempt to fit the problem to the tool (for example, using integration by parts to solve a second order differential equation.) When forced to justify their work, they often see their errors without further prompting. Even when a student has done the work correctly, this question encourages them to review and analyze rather than to rely solely on the instructors approbation.

4. *Why do you think you performed poorly?*

This question, almost always used in a counseling setting, asks the student to examine not only their mathematical work, but also their overall personal and professional lives. By encouraging students to think about a poor quiz or test score, the teacher encourages them to assess not only their academic ability, but also such skills as organization, planning, and so forth. This type of self-assessment is especially important to encourage because the instructor cannot see all aspects of a student's life and therefore is often unable to adequately assess the causes of a poor performance. When grading one particularly poor project, I appended a comment that to me it seemed clear that the two cadets had not put very much effort into their work. Upon reading my comments, one of the two assured me that they had started early and put a lot of time into the project. When I asked him why *he* thought they had done poorly, he was able to cite several causes more specific than "little effort." While I could suggest ways to deal with the sources of a bad performance, he was in a better position to identify those sources. Furthermore, without being asked the question, many students will simply accept a poor performance without examining the underlying reasons.

We often discuss assessment as if it were the sole proprietary domain of the instructor – to determine if understanding has been achieved, to give feedback, to assign grades, and so forth. Certainly each of these is necessary and important *per se*. But they can also be instruments to teach our students how to assess themselves. It is not enough to inform our students (or ourselves) that they do or don't have understanding of the subject matter. We must also help them to see why they do (or don't understand) and ultimately, we must help them to be able to make such a determination themselves.

Student Growth Assessment at USMA

LTC Steve Horton, USMA

The Department of Mathematical Sciences at West Point has established *learning thread objectives* that we use as a framework to help us measure the growth of our students. These objectives fall in to five categories: mathematical reasoning, mathematical modeling, scientific computing, communicating mathematics, and the history of mathematics. Each of these categories contains several objectives. For example, the mathematical reasoning category includes the objective "recognize valid and invalid logical arguments." A complete list of these objectives appears in our Core Mathematics at USMA handbook, which is available on request.

The Department teaches four core mathematics courses to each class of cadets. Most cadets take these four courses in their first four semesters at USMA. We expect cadets to grow as students during these four semesters. It is the assessment of this growth that is the topic of this article. It is our experience that measuring student growth in terms of these learning threads is difficult. Measuring their growth in terms of *content* is easier; most traditional methods (exams, homework, etc.) do this. We believe assessing their growth as learners, however, requires a somewhat different approach. For several years, a method to measure student growth in our learning thread objectives has been evolving in the Department of Mathematical Sciences. We start by looking at what has been done in the past.

One of the assessment methods we experimented with in previous semesters involved "growth essays." These were essays written by cadets, typically one essay per growth objective category per student for each core course. This was tried both as out-of-class assignments and as in-class exercises. The idea was to keep track each student's answer to each essay over his or her four semesters with us in order to obtain evidence of growth. This approach proved difficult and time consuming. The typical instructor had the additional burden of assessing 250 additional essays (5 for each of 50 students) in an already busy semester. Keeping students' essays from previous semesters and then comparing them to their current semester efforts was also a challenge. In addition, a significant amount of cadet time was consumed writing these essays which reduced the total time available for them to actually do the learning that we were trying to measure! In the end, although some interesting anecdotal evidence of growth was obtained, it was decided to abandon this idea in favor of something less burdensome.

The initiative to change our growth assessment method came not only from the difficulties involved with the old system, but also from the observation that we were really more interested in the growth of the *class*, as opposed to that of individuals in the class. The system that we are now experimenting with involves minimal additional instructor effort. Rather than create new requirements such as growth essays, we simply ask the instructors to fill out a form like that shown in Figure 1 for each major course project the cadets turn in. The instructor simply circles E, S, or N for each category and adds comments where appropriate.

Learning Thread Assessment	Comments
Mathematical Reasoning	E S N
Mathematical Modeling	E S N
Scientific Computing	E S N
Communicating Mathematics	E S N

E – Excellent S – Satisfactory N – Needs Improvement

Figure 1 Learning Thread Assessment Matrix

This small effort provides two significant benefits. First, instructors use this form to assess student growth in the learning threads[1]. They then use the information gathered to contribute to the program director's assessment of the class as they pass through the current course. Although this assessment is clearly subjective, the program director, who typically serves for several years in that position, can compare student learning growth from class to class, and thus get a general idea if the Department's programs are "on the right track." The other benefit is that the cadets see the learning thread assessment matrix and their instructor's assessment, and recognize that we care about the growth of their learning abilities. The questions they have asked about this system have generated some interesting discussions that probably would not have otherwise occurred.

As has been mentioned, this is minimally burdensome to the cadets and instructors. Cadets simply do their projects, as they would be doing if we were not doing anything to assess their learning growth. The instructors fill in the learning thread assessment matrix which is easy to do at the same time they grade the project. They then report their results to their program director.

We make no claim that this system is perfect. The process is subjective and difficult to calibrate. However, if we espouse the importance of the learning threads as major objectives in our mathematics program, we must make an effort to assess our program against these objectives. We will continue to refine our growth assessment process in an attempt to gather the highest quality of information about our progress in student growth with the least amount of impact on our (already fully engaged) faculty.

Attempts at Gaining Insights into Student Understanding

LTC Kathleen Snook

Accurately assessing students' understanding of mathematical concepts is a difficult task. To obtain a more comprehensive assessment of students' understanding, it seems instructors should use a variety of sources, each of which incorporates a variety of problem types and presentations. In a recent study investigating students' understanding of the concept of the derivative, a talk-aloud problem solving interview technique allowed students more opportunity to reveal their depth of understanding than did a written instrument. In some cases students indicated a deeper understanding in the interview than on the written instrument, while in other cases a student's procedural proficiency on the written instrument masked a lack of understanding evident in the interview. The technique of interviewing for assessment purposes is a time consuming and difficult task in our already busy environments. Yet, we are charged with accurately determining (to some degree) each student's level of understanding

[1] The history thread is omitted since the history of mathematics does not generally appear on each project. Other means-such as short essays or student presentations-can be used to evaluate student growth in the history thread.

and ultimately assigning him or her an appropriate grade. We may wonder how variations in classroom activities and assessment instruments would impact our perception of students' levels of understanding? It seems plausible that if students are exposed to many different types of problems, and if they have opportunities to discuss and verbalize solutions, then instructors may ascertain from these activities an accurate picture of a student's level of understanding.

At USMA the core courses have incorporated interdisciplinary applications for several years. Small classes are taught in an interactive manner with a focus on applications of the mathematics under study. The Department of Mathematical Sciences emphasizes use of applications and varied assessments of students' understanding. In addition to instructor quizzes and course exams, applied classroom problems, computer mini-projects and major projects are assessed components of the courses (some graded and some not graded). In-class activities and discussions provide instructors with a close equivalent to "talk-aloud problem solving" in order for them to assess students' levels of understanding. Outside of class projects offer opportunities for students to display their understanding by solving applied problems and analyzing their solutions. These projects require students to conceptually and procedurally tackle an interdisciplinary problem by developing a mathematical model, solving their model, and finally analyzing their results. Students submit a written report and instructors assess the quality of students' mathematical solutions, as well as their analyses.

Assessment of these varied activities has been fruitful in determining students' levels of understanding. Exams and quizzes consist of both procedural and conceptual problems. They are designed to include a variety of problem types and presentations. Exam authors design problems from which they can make inferences about students' understanding. Classroom problem solving activities generate discussion about the concepts and procedures needed to solve applied problems. During those discussions, instructors are able to assess understanding and address misunderstandings. Projects done outside of class offer a wealth of information about students' procedural and conceptual understanding. These projects allow not only assessment of concepts and procedures for a particular course, but also offer insights into student growth (see related article by LTC Steve Horton). The desired result of these assessments is that instructors are able to confidently assign to each student a grade that reflects accurately that student's level of understanding of the course material.

Some Essential Ingredients for Effective Assessment

Edward Connors and Mary Ann Connors, Ph.D.'s, USMA

In order to effectively assess students' mastery of course material, it is essential to prepare them to be at the top of their game; that is, to be in a position where they are well disposed to give their optimal performance. We provide below a personal perspective on some components for student assessment in the context of the core mathematics courses at USMA, West Point. The components are comprised of interactive class activities including student briefs on solutions to problems, traditional quizzes and exams, collaborative projects, and students' reflections on their perceived growth in understanding concepts.

Variety is not only the spice of life, it is a vital constituent in the classroom. Instructors strive to be effective leaders and coaches by posing problems, questioning, suggesting, directing, and providing brief lectures when necessary. Cadets give briefs on solutions to problems for a significant amount of class time. This format provides numerous occasions to take snapshots of student progress. Moreover, it enables the cadets to hone their communication skills and prepare for more formal assessment.

The mathematics department implements a teaching method named for Sylvanus Thayer, Father of the United States Military Academy. The Thayer Method embodies student active learning. It requires that the cadets take responsibility for their own learning. They are expected to read the lesson on the daily objectives, which are clearly stated in their course guide, and to complete problems before class. When class begins, an interactive discussion on the lesson ensues. Some students brief on the concepts and exercises in the assignment. These may contain a variety of interdisciplinary applications sometimes including a military scenario and requiring appropriate use of calculator and computer technology.

Usually students are then presented with a set of board problems. Often they work at the board in teams of two or three discussing and writing the solutions. (Mathematics classrooms are equipped with eighteen boards, one per student.) While moving around the room, the instructor witnesses and assesses the learning experience. The interaction of the students is dynamic and usually productive. The goal is to encourage independent thinking, which results in finding a common solution. Students solve problems in different ways. Sharing ideas on problem solving increases the scope of their problem solving skills and gives them a better understanding of the breadth and depth of mathematical reasoning.
Students present briefs on their solutions to the problems. Every brief ends with, "Are there any questions?" This often leads to some discussion or, at least, short answers. This provides immediate accurate feedback to the instructor and the students on the status of their understanding. If students require additional instruction, they are encouraged to consult the instructor. Much of the communication between instructors and students occurs by e-mail and instructor web pages. The communication enables the instructor to assess the current status of students' understanding.

Traditional assessments in the form of written quizzes and exams including a cumulative final exam are administered. Questions are in free response form. Students are required to show all their work. Some questions may require them to justify their answers or explain the process. They are required to answer in complete sentences in short essay type questions. Great care is taken in writing appropriate questions testing comprehension of lesson objectives. Writing good exam questions that adequately and fairly assess students' mastery of stated goals is a challenge. Each exam is crafted by a different team of instructors.

Projects (usually three) are required each semester. A very challenging applied interdisciplinary problem with several parts is assigned. The problem is worded to convey the impression that the cadets are participating in a real experience similar to a military mission. It usually requires mathematical modeling. The cadets are required to write an executive summary, as experts reporting to a commanding officer, and provide full documentation of their work as parts of the project. It is required that the students complete the document using state of the art technology. The document must be typed. Graphs and mathematical calculations are done with appropriate computer software and inserted in the paper. Students collaborate in groups of two or three. Occasionally individual projects are assigned.

Projects provide a different dimension in the assessment process. Students are working together on a problem that is not completely laid out for them. They are required to analyze the problem, do research, if necessary, make decisions, and find results. Sometimes they are asked to make recommendations based on their findings. This provides an opportunity for them to interpret their results as it relates to real life.

Reflective summaries are submitted three times each semester (usually after an exam). Students reflect on what concepts they learned and share their personal reflections on their growth. They often comment in depth on the positive features of the different forms of assessment discussed in this paper. For example, some students explain how solving the board problems with a partner and briefing helped them to understand the concepts and procedures better. Some discuss how the projects helped them to understand better and also to recognize the importance and relevance of the mathematics studied in the course. Some report a sense of accomplishment and personal pride, particularly, on individual projects.

Assessment is a bilateral process. The students' self-assessment in the reflective summaries provide important and useful feedback to the instructor. Students are also required to complete an anonymous electronic course feedback form on the internal web. Their evaluations assist instructors in assessing how well the course goals were accomplished.

As instructors, we believe that these varied snapshots of students realizing course goals present us with a broad composite of student achievement. We are confident that the assessment is fair and accurate (see related article by LTC Kathleen Snook).

(Comments welcome:
edward-connors@usma.edu
maryann-connors@usma.edu)

Teaching the Test (and Other Alternatives to Traditional Assessment)

Capt Frank Wilson
US Air Force Academy

Throughout our educational experience, most of us have been exposed to traditional evaluation and assessment paradigms. There are strengths in traditional methods, but sometimes a more maverick assessment approach is necessary. Assessment techniques can be improved by grading knowledge levels (not learning rates), clearly identifying testable objectives, and providing constant remediation.

Grading Knowledge Levels (Not Learning Rates)

Grades are intended to represent a student's level of knowledge; however, many times we grade students on their learning rates rather than on what they ultimately learn. Let me illustrate:

Suppose Jane Student earns the following exam scores: 57/100, 63/100, 85/100, 76/100, 182/200. Her overall test average is 77% but her final exam score shows that she has mastered 91% of the material. Unfortunately, most of us would not give her the "A" grade. We would penalize her because she did not learn the material quickly. In this case, her grade would not represent her ultimate level of knowledge. There is a better way.

As an undergraduate, I initially struggled in a complex analysis course. After I failed the first exam my professor called me into his office. Since I was also the grader for another course he taught, I thought he was going to fire me because of my poor exam performance. Instead, he expressed genuine concern for my academic welfare, and we discussed how I could become successful. Then he said, "Let's just drop this exam score." I got a "C" on the next exam and, by the final exam, I had reached a "B" level of knowledge in complex analysis. The "B" posted to my transcript accurately represented my level of understanding in complex analysis—not my initial incompetence.

Some teachers may be concerned that giving students the option to take their final exam grade as their final grade may reduce student motivation during the course. What's to keep students from blowing off their studies and cramming the night before the final? Students do not need to know about the final grade option at the start of the semester. By revealing the option midway through the semester or on an "as-needed" basis, struggling students who are on the verge of giving up can be motivated to redouble their efforts in hopes of scoring high on the final exam.

Clearly Identifying Testable Objectives

Oftentimes teachers and students play a game of cat-and-mouse when it comes to exams. Students try to extract as much information from their teacher as they can about a forthcoming exam while the instructor tries to withhold information in an effort to maintain an element of surprise. But if our focus in teaching is student learning, is it necessary to withhold information?

While enrolled in a graduate-level instructional design course, I had a professor who didn't think so. Prior to the final exam, the instructor gave us a list of all of the potential exam questions. This list clearly emphasized the important objectives of the course. In preparing for the exam, diligent students ensured that they had a clear understanding of the answer to each and every question and the associated objectives. The exam was oral and consisted of a single question for each student. The entire class convened, and one-by-one the professor called on each student to respond. In the process of the exam, the key objectives were verbally reiterated as the students responded to their teacher's queries. If the professor was uncertain about the depth of a student's understanding, he pressed the issue with a series of follow-up questions until he was satisfied that he had accurately assessed the student's understanding. Not everyone earned an "A," but everyone did know what objectives had to be mastered to earn an "A." Each student's grade directly reflected that person's diligence in mastering the clearly-identified course objectives. Could such a teaching model be applied to mathematics?

If we are truly focussed on student learning, we need to make sure our students clearly understand the course objectives and the types of problems that we expect them to solve. In many math courses, we present some material that we don't expect the students to master. Our intent is to introduce them to material that they will eventually be required to master in future courses. There is value in this approach; however, the student may have difficulty discerning the topics we expect them to master. We can clarify what topics require mastery by periodically giving students comprehensive review worksheets that illustrate the types of problems we expect them to solve. Critics may consider such actions as "teaching the test." But is "teaching the test" a bad thing? Although I do not advocate giving out the exam in advance in an undergraduate math course, I believe it is essential that we give students a well-defined idea of the types of problems and level of comprehension we expect. The element of surprise is critical in war but is not necessary when assessing student academic understanding.

Providing Constant Remediation

Most of us validate our learning by receiving feedback from an expert. Frequently, subject mastery doesn't occur after a single feedback session. There is a continuous flow of feedback that shapes our ability to think and, consequently, to perform. As teachers, we typically provide feedback with red ink on a graded event. Rarely do we give the student an opportunity to respond to our corrections. How well does this type of feedback model what students will be faced with when they enter active duty? Most tasks in the real world include many opportunities for feedback and multiple iterations.

In the aforementioned instructional design course, the professor allowed students to resubmit graded assignments multiple times. For example, if a student earned a score of 15/20 points on an assignment, he could resubmit it after correcting errors identified by the instructor. The instructor would review the student's revisions and adjust the score accordingly.

The thought of rescoring student assignments may be formidable due to heavy teaching loads and administrative responsibilities, but there are tools that can ease the burden. For example, in the Calculus I course at USAFA, students complete a series of differentiation problems via the *Dif* computer program. Students are required to complete at least 20 problems but may continue to do problems until they are satisfied with their score. The program instantly provides feedback on their performance. Although it would be even more effective if it provided feedback on why they missed a problem, the program itself is a step in the right direction in providing continuous feedback and remediation.

Why not allow students to turn an assignment in a few days early to receive written feedback? This practice results in a little extra work for the instructor, but allows an opportunity for remediation and an improved grade.

Perhaps the best feedback dialogue occurs in one-on-one discussions during office appointments. These events allow teachers to diagnose individual academic weaknesses and to provide detailed training on how to think analytically. Although extremely valuable, only a small number of students participate in one-on-one academic training with their instructors.

There are other methods to provide frequent remediation to our students. As teachers, we should continually look for opportunities to provide feedback and reward students for positively responding to it.

As we focus on grading knowledge levels, clearly identifying testable objectives, and providing constant remediation, we should see an increase in our students' performance. Ultimately, they will learn better and learn more. Isn't that what teaching is all about?

The Quiz as a Learning Tool

LTCs Jeff Appleget and Kevin Pilgrim, USMA

To many students and instructors alike, the onset of a quiz is a signal that the learning for a particular block of instruction has been completed, and it is time for a quiz. Inherent in this thought process is the implication that the sole purpose of a quiz is to evaluate how well the student has assimilated the required material. Most would agree that the threat of a quiz might prompt some learning to occur before the student sees the quiz, but not many of us envision that a quiz can help the learning process once the student has the quiz in his or her hands. We will discuss two non-traditional methods of quizzing that foster the learning process.

Publishing the quiz the night before

Discussion

This technique provides the students with a copy of the quiz that they will take the next day in class. It is a great technique for freshmen math students. Experience has shown that the typical incoming students spent 15-30 minutes at most preparing for a high school math lesson; they seem to come from a "spoon-fed" learning environment. As their first college math instructor, one critical task that must be accomplished is to wean students from the habit of coming to class expecting to be spoon-fed. By providing the student with the weekly quiz the night before the quiz will be administered, all doubt is removed as to what food they must digest. At the same time, the finding of the spoon and the actual feeding is left to the students. The students begin to be responsible for their own learning.

A by-product of this technique is the maturation process many students experience in the area of collaborative learning. High school students are familiar with completing required homework assignments in a group around a study hall table, but the task of preparing for a published quiz is inherently different. Instead of bringing a paper copy of their homework to class for submission, they must bring the *knowledge* of how to solve the required problems to class. While not all students need the help of their classmates preparing for the published quiz, many students create email chat lines and seek out classmates to discuss the merits of various problem solutions. Students within walking distance often congregate in the halls to try to figure out "the quiz." Students start to learn how to assimilate knowledge from fellow classmates rather than copying down a classmate's solution for inclusion in a homework submission.

Interestingly, the quiz averages are no different from the instructors who give the traditional unannounced or announced quiz where the quiz is revealed to the students in class. Typically, the student with average mathematical aptitude but a strong sense of duty and good work ethic does about a letter grade better than the mean on published quizzes. The mathematically gifted student that spends little preparation time usually achieves the mean, and the unmotivated or inept students typically still fail.

Allowing the students time to ask questions in class immediately before administering the published quiz also provides an opportunity to learn. Students usually enter class the day of the quiz discussing the approaches to one or more of the quiz problems. At first, they ask indirect questions that touch on how to do a particular problem. Diverging from the nuts and bolts problem-solving details and refocusing their question at a conceptual level forces them to demonstrate they understand the problem. Working an example problem similar to one on the quiz is always met with the rapt attention of the students. Because there are multiple ways to solve real world meaningful problems, this "hand out the poop" teaching period is usually focused on the conceptual knowledge of the particular material being tested.

Tips and Techniques

Quiz problems are taken from a variety of sources and are above average in difficulty when compared with those given in-class by other instructors. All quiz questions are tied directly to lesson objectives. All but one will focus on the previous 2-3 lessons, and one problem usually comes from that day's reading. One problem will require a thorough conceptual understanding of a key lesson objective.

To make sure the goal of collaborative learning does not degenerate into a system where a bright student works the quiz and then publishes the 'approved solution' for short-term memorization by the less-motivated students, various adaptations to publishing the actual, verbatim quiz the night before can be made. One technique is to publish multiple quizzes to the website (three seems to be a good number), from which one will be chosen for record the following day. For the students to work all three quizzes in order to have the 'approved solution' would be a significant time investment, so most optimize and try to learn the concepts behind the problems that will be tested.

Another technique is to just change the data from the single published quiz so that the problem is exactly the same, but the numerical answer will be different. Again, this precludes the students from writing down solutions they have memorized and trying to reverse engineer the answers.

An interesting deviation is to shift the quiz from the published quiz to a subset of the published questions along with questions from suggested homework problems and/or textbook example problems. This quiz is given as an "open book, open notes" quiz. This helps emphasize the need to keep up with the required homework, and separates the students that prepare for class each day from those that only study for the quiz.

Finally, a hybrid of many of the above techniques can be produced by publishing essay and true/false questions, and augment those with (unannounced) assigned homework or significant textbook example problems.

Summary

Instructors are role models. Instructors at West Point should be role models for leaders. Good leaders communicate standards and a vision of what is important. Poor instructors do not help their students separate the "wheat from the chaff." Students and soldiers like to know the standards and be given a fair shot at obtaining them. Learning is inefficient but evaluation should be a means to reinforce the important and not uncover the obscure.

Student-graded quizzes

Discussion

Another technique that blends evaluation and learning is the student-graded quiz. A quiz is administered in class in the traditional manner, but instead of collecting the papers to be graded by the instructor at a later date, students exchange papers and grade each other's work. The first learning opportunity occurs as the student is exposed to a classmate's problem-solving approach. Just as an old axiom says that more is learned through failure than success, a student really learns more when grading a good attempt at a problem that falls a little short than grading the textbook answer. (So go ahead and make the quiz challenging!) Finding the mistakes and seeing why the mistakes were made teach important lessons about problem-solving methodology. Of course, not all quizzes will be good attempts. Some students will gain an appreciation for how hard assessment can be when a fellow student uses pen to do a math quiz, writes in a seemingly foreign language, or is just generally sloppy and disorganized with his or her work. Still others will witness the result of not preparing for a quiz.

Learning also takes place in the discourse about partial credit that inevitably occurs during the in-class grading. Students often complain when they are returned an instructor-graded event--"Why did I lose all these points when it was (as it invariably always is) 'almost' right?" Since the awarding of partial credit points will parallel the building blocks of the solution process for a particular problem, the debate about partial credit that most instructors loathe now becomes a great opportunity to discuss the skills and concepts a particular quiz problem is attempting to evaluate. Of course, you must break the ages-old unwritten rule of never allowing a student to see the cut sheet!

Grading a quiz in class also provides immediate feedback for the students, a cornerstone of the assessment process. In fact, if you have students exchange papers with someone sitting beside them, you can witness a student grading a classmate's quiz while also carefully (and actively) monitoring the grading of his or her own quiz by the adjacent classmate.

Tips and techniques

The grading of the quiz by the students may take nearly as long as the time provided to take the quiz. The partial credit discussions that occur are usually the reason, but as alluded to above, that is where much learning takes place. In order to control the amount of time spent grading, it is essential that you give a lot of thought to the amount of partial credit that will be awarded for each problem. No matter what you do, you'll never be able to anticipate all the twists and turns that a particular student's solution process will take! Remember that the goal is learning, not assigning a grade. If the students think a particular cut is somewhat unfair, give them back some of the points! If a student can't figure out what credit should be awarded, tell the student to give the classmate the benefit of the doubt. Make sure that you emphasize what particular skill the problem is evaluating and the correct solution technique(s)--that is what the students should leave class with. If they escape with a few "extra" points, the mathematical world, as we know it, will not implode. You may want to collect the quizzes and do a quality control check on the grading. Again, this is not to ensure that students don't get "undeserved" points, but rather to ensure that the quiz that you will return to them doesn't indicate that an incorrect solution process was evaluated as being correct by a lenient or inattentive grader. Once in a while, you may actually find a cadet that grades more severely than you would. In that case, an upward adjustment of the score may be in order. Never adjust scores downward; the points are not important! If you want to avoid collecting the quizzes at all, and just pass a mark book around to record the grades, you should hand out, or publish to the course website, the complete solution. Again, this is to ensure that the cadet has the correct solution methodology available to use for future studying.

Summary

Student-graded quizzes can be a great learning tool. Instructors have to break out of the paradigm that a quiz is solely a tool that assigns a grade to a student's demonstrated ability, and focus on the learning that should occur. In most cases, it doesn't save a lot of time for the instructor since more thought must be put into the structure of the quiz and the cut sheet needs to be prepared in advance and in finer detail. You also will use more class time for the grading, but it will not be "wasted" time if learning takes place.

After Eight Years of Reform, a Steady State Compromise?

COL Richard West, USMA

In 1990, West Point implemented a bold new curriculum of core mathematics for all of its students. The new curriculum builds on an initial course of discrete dynamical systems that includes matrix algebra and the analysis of systems of equations, followed by a two-course sequence of lively calculus that includes differential equations, vector calculus and multivariable calculus, and concludes with a course in probability and statistics. At the time of implementation, the Department of Mathematical Sciences, with help from others, outlined its expectations or goals for student growth after this four-course, two-year program -- a true seven-into-four curriculum. After a year of this new program, we saw a need to create formative objectives within the courses that lead to these summative goals. The conceptual framework for articulating these objectives took the form of five educational threads that weave themselves through the content of the four courses and beyond. These threads, the subject of a paper presented in 1992, are mathematical reasoning, history of mathematics, scientific computing, mathematical modeling, and communications in mathematics. For each of these threads, we further refined goals and formulated objectives which sequentially improve on the previous term s objectives. In 1993, 1994, 1996, and 1998 I presented papers on my evaluation of this implemented curriculum reform and some of the lessons learned about this ongoing longitudinal study. After seven years' use of these threads and eight years of the new curriculum, we are convinced of the worthiness of both endeavors. Data and interviews were collected for both the original reform cohort and a comparison cohort that covered a more traditional core mathematics curriculum. Both of these cohorts had about 1000 students and both have graduated. The results indicated that the reform cohort performed as well as or better than the comparison cohort in mathematics tests, mathematics courses, or common mathematics-based courses such as physics and engineering science. Also, the reform cohort's attitude toward mathematics appeared to have increased positively over the first three courses and remained stable through the fourth course. Data continue to be collected for seven more treatment cohorts of about 1000 each. Indeed, changes continue to be made as this reform moves through its eighth year. This current year three new texts were selected for three of the four courses. The latest changes were brought on by the impacts of reform texts on faculty preparation time. We continue to fine-tune assessment instruments from term to term, and analysis and database management are set up to do real time evaluation that could facilitate between course and midcourse curriculum and teaching adjustments. Longitudinal results on performance and attitudes toward mathematics will be reported.

EVALUATION AND ASSESSMENT IN THE CLASSROOM

LCDR Sal Ceraolo

Evaluating and assessing student comprehension of material gives rise to many methods, personal preferences, and styles for classroom homework, quizzes and testing. The time demands placed on a midshipman necessitate some form of weekly evaluation to ensure that they are devoting an adequate amount of preparatory time to mathematical topics. In addition, various means of keeping midshipman actively engaged in the classroom learning process are employed.

Homework is the first evaluation method explored. Throughout the Math Department, homework is considered essential. Some instructors feel the need to assign more than the syllabus required problems, as they believe understanding comes with the constant reinforcement or practice provided with repetition. They may collect homework to see if it was attempted or just quiz from homework problems. One instructor will call a different midshipman to the board to go over each homework problem at random. The homework will give the midshipman an indicator if a problem exits that may require extra attention. All instructors require midshipman to produce homework attempted when scheduling Extra Instruction. The general consensus is homework is good, more is better.

Quizzing takes on many different varieties. Many instructors who do not collect homework will give quizzes that are taken directly from homework problems that were assigned and went over in class. This will also indicate who pays attention in class. Some instructors will modify homework problems or place a mixture of homework and modified problems on the quiz. One instructor gave a few group quizzes where the students worked as a group to solve the problems and had a spokesperson present it to the class. All instructors agree on quizzing every week from 15 to 20 minutes, whether it be announced or unannounced. This provides timely feedback to the student and indicates weak areas to the instructor.

Testing is a necessity and also takes on many formats. Tests can be made up from modified homework problems or exercises not assigned. Old tests and other Calculus texts are also used when designing test problems, which cover concepts taught in class. Many of the instructors do not believe multiple choice tests work as many students will feel time constrained and guess or do a problem on the calculator, like the TI-92, and then the instructor is not certain whether the material is understood. With a short answer exam, partial credit can be given and the instructor can follow the student's logic train used to achieve the answer.

During low attention span periods, like Army—Navy Week or prior to a holiday, some instructors will give out work sheets with problems worked out and material covered in class on the sheet. This will allow the student to have the material available for review at a later time. Many instructors actively engage students in classroom discussion through board work or group work. The instructor can also use this to assess the progress of the midshipman in understanding the material.

The bottom line through all of this is to provide timely feedback to both the instructor and the midshipman as to what areas require added attention or re-emphasis. Constant repetition and quizzing are the keys to success in Mathematics.

It Worked – I Think

LTC Gary Krahn, PUSMA

We have all heard the adage, "You can lead a horse to water, but you cannot make it drink." I reconstructed that adage during my years as an artillery officer to, "You can lead a horse to water, but you cannot make it drink; however, you can make it wished it had." My transition from training to education provides a new perspective on this "old" adage. As educators of lifetime learners, it is not about leading a horse to water or even making the horse drink. It is about making the horse thirsty. There are clearly an uncountable number of factors that create thirst – from leadership to content. Work, however, seems to be the best way to create thirst!

I assign three different types of work to be accomplished outside of class: Suggested Problems, Drill Problems, and Homework. Suggested problems are exercises to hone procedures and explore concepts. A cadet has no responsibility to complete suggested problems. Drill Problems are a duty. If a cadet cannot complete the drill, they are expected to notify me in the same manner I would expect a lieutenant to notify me if an assigned mission could not be completed. Homework is similar to drill, however, documentation now becomes a factor. I check drill problems now and then in the same spirit I would check the 2/17 FA motor pool. I don't need to check everyday unless it requires more attention. This "style" of leadership has always been effective in promoting individual responsibility. Before I assign drill, I try to determine what concepts and procedures need to be inspired, encouraged, or energized before the cadets begin their responsibility to complete the assigned work. I assign several drill problems during class to be completed in the future. Before departing class, however, I have students or myself attempt the first couple problems to share the concerns and insights about the problems. During this "scaffolding" process, problem solving techniques and concepts begin to spew out. More importantly, cadets take ownership and tend to accept their responsibility for learning with greater confidence. It is fun to note the similarities between being an officer and a teacher.

"I can't understand why people aren't just dying to learn, why it isn't the greatest adventure in the world – because it's the process of becoming. Every time we learn something new, we become something new." Leo Bascoglia.

When Students Let You Down

LTC Patrick J. Driscoll, USMA

You've worked days, maybe weeks, crafting the perfect interdisciplinary project for student groups in your course. It ties together many of the course topics, and extends these ideas into areas that illustrate their generalization well beyond mathematics. In fact, you're actually excited because you've kept the problem simply posed and allotted plenty of time in order to enable them to submit a creative, professional effort. And, they have the technology and the knowledge to do so.

Then, sha-bang! Reality comes knocking at your door like the Grim Reaper. Three groups of seniors whom you were really counting on doing well turn in projects that look like they bought them from some fly-by-night, web-based, grade-school project service. Little, if any, analysis; major parts missing; wrong format, etc. What to do?

Once you get past considering the truly vitriolic and medieval options, you settle down and reflect on the fact that despite all you do to motivate these folks towards academic excellence, it is *their* time management processes and personal prioritization methodology that matters. However, they are going to become officers in some branch of service, and therein lies the key to what I've instituted in my Nonlinear Optimization course to handle the situation described earlier (which happened this semester).

This inspiration came from my experience with the U.S. Army Ranger School, although it has commonality with other specialty courses, as well. When an officer gets injured during one of the phases of Ranger School, they are offered the opportunity to "recycle" that phase after recovering from the injury. And, if the injury repeats itself later, this recycle opportunity is simply repeated over again. Thus, there is always a small handful of officers who graduate from Ranger School as "Ra-Rangers" (once recycled), or Ra-Ra-Rangers (twice recycled), and so on.

Although this practice requires the officer to repeat an entire phase, which can be a bit disheartening if the injury occurred near the end of the phase in question, it places the burden of responsibility for successful completion back on the officer, where it belongs, and not on the school. The officer can opt out of recycling by signing a document that acknowledges the opportunity they have been offered and their explicit statement of turning it down. Not a good thing to do if you value your career.

I modified this approach for use in my course, maintaining the essential ingredients but tailoring the objective to focus on our academic environment. A copy of the "contract" follows. The nice aspect of taking this approach is that it meshes well with USMA's efforts to integrate Army practices and standards into cadet leadership development.

After first failing the projects for obvious below-standard effort, this technique allows students to recover a proportion of their points lost, with the potential of adjusting their failing grade to a maximum of a C (75%) The key is to not allow students who accept this opportunity to do better than others who worked hard on the original project. I offered this opportunity to any group receiving less that a 75% score. The groups decided whether to participate in this opportunity based on their potential marginal gains versus the anticipated effort required. Consequently, a group who received a 73% on their submission could only improve their grade to a 75% if they received a 100% on the remedial project. So they would more than likely not participate.

This approach was well-received by the student groups. They considered it to be very fair and appreciated the opportunity to repair the damage to their grade point average that *they* had caused. Without exception, they admitted that it was their poor time management planning and underestimation of required effort that caused their earlier substandard submission.

The qualities I find appealing with this approach are: (1) it puts responsibility on the shoulders of the students, where it belongs; (2) it recognizes the developmental nature of both academics and leadership; and finally, (3) it allows students a limited, reasonable second chance but requires them to work for it.

Purpose and Intent. This special project is designed to provide those project groups scoring less than a 75% overall on the Unconstrained Optimization Project the ability to improve their grade to a maximum score of 75% (225/300) from their current score. Your score on this project will be used as a multiplier using the formula $(0.xx)\cdot(\Delta projscore)$, where the first term is your grade on this project, and the second is the difference between your current project grade and 225. Because it is an optional project, its completion is considered "in addition to" normal course requirements. No compensation time or class drops will be given to facilitate its completion. All groups being offered this option must complete the following section and return it to your instructor in-class **Wednesday, 18 November 1998**.

1. I understand that this project is being offered as an optional means of improving my Project score, which is currently below 75%. The due date for this project is **NO LATER THAN 1600 hours, 25 November 1998**. I choose:

☐ NOT to accept this option

☐ To accept this option

Name: ____________________ Signature: ________________________

☐ NOT to accept this option

☐ To accept this option

Name: ____________________ Signature: ________________________

The purpose is to assess your group's ability to do two things:

1. Correctly the Method of Steepest Descent, and
2. Solve an optimization problem using the Kuhn-Tucker Optimality conditions, and analyze and interpret mathematical results.

Excessive collaboration outside of your project group in the opinion of the graders will result in overall grade penalty. You may use any software at your disposal to answer the questions posed. Visualization will aid significantly in the analysis you need to do. The combination to the EECS/Math MAPLE labs will be made available to you.

Requirements. Write a well-organized, logical technical report to answer the questions posed in what follows. Sufficiently demonstrate, show, justify and explain all of your results and conclusions.

Seven Years and Four Math Departments Ago

Dr. Ted Stanford, United States Naval Academy

It was seven years and four math departments ago that I first had to sit down at the end of a semester and assign grades to a class full of students. Still, every time I'm faced with this responsibility, I ponder and agonize: What exactly do grades mean? What exactly am I evaluating, and why? I know some answers to these questions, of course; and I use exams and points and grading scales in the same ways that many of you do, but I still wonder to what extent the grades I assign reflect something meaningful about my students and what they have learned. I used this article as an opportunity to talk to some of the other math instructors here at USNA about their ideas on grading. What follows are my own reflections together with some of the input from my colleagues. It is my hope to continue to discuss these issues, both with the members of my department and with any of you at the other academies who are interested. I'd be glad to hear any of your responses.

My primary goal as an instructor is to teach my students to think mathematically. Many students, however, are not particularly interested in thinking mathematically. Most students are interested primarily in getting the best grade they can with the least amount of effort. (I do not mean that as a negative judgment on students.) To the extent that they are interested in learning some mathematics, many of them simply want to know "how to do the problems", meaning that they want to know a specific list of procedures for solving specific types of problems. I see my task, then, as some kind of compromise between my goal and theirs. I show them how to do some of the standard types of problems, and give them opportunities to earn points toward good grades if they learn how to do those problems well. But sometimes I require them to exercise their higher mathematical thinking skills on more challenging problems. I also look for ways to introduce mathematical thinking into class discussion, even when their grades don't depend on it. Although the students' first concern is their grades, I find that some of them appreciate a certain amount of theoretical explanation of what we are doing, and even a digression now and then into some mathematically interesting topic that is only slightly related to the problems we are supposed to be learning how to do.

I see two main purposes for grades. The first is as a means of comparing students. Company officers, service selection boards, graduate schools, potential employers, and others need to be able to rank students. Also, as one professor pointed out to me, it is important that the students have a way to compare themselves with their peers. Another professor pointed out that even though grades will be used to award opportunities to some students (and deny them to others), the instructor assigning the grades should not think too much about that, since this results in pressure to raise borderline grades and can lead to overall grade inflation.

Since students are ranked by their grades, a certain amount of competition develops, and leads to the second main purpose (in my opinion) of grading, namely that grades are a motivation for students to study. I see grades as my carrot and stick for getting students to spend time and effort on my course.

A number of my colleagues felt that grades are also very important as an indicator of how well students have learned the material in the course. Personally, I find this to be one of the more problematic aspects of grading. For starters, there is the problem of determining exactly how well you expect your students to know the material. Given a set of homework problems on which to base an exam, it is often possible, within what I consider a reasonable and fair range, to write an exam that nearly everyone will get over 90 percent on, and it is also possible to write an exam that almost nobody will get better than 90 percent on. Expectations will also vary quite a bit from instructor to instructor.

Furthermore, there is a fair amount of randomness to what is included and what is not included in even a "standard" calculus text or syllabus. In my view, the most important thing is that studying the material should provide students with the opportunity and motivation to exercise their mathematical thinking and reasoning skills. Which specific topics are chosen for this purpose is mostly of secondary importance. (One exception to this is that if course A is a prerequisite for course B, then obviously the students need to learn the things in A that will prepare them for B.) Most people, even in technical fields, do not remember most of the specific topics that were covered in their "three-ness."

In order for grades to be fair, there needs to be some consistency from one section and instructor to the next. I taught for two years at a large public university in California, and not once did I get any feedback from the department as to whether my grading was consistent with what other instructors were doing. On the one hand, I did appreciate being allowed to run my own show. On the other hand, I had no way of knowing whether I was being fair to my students, and whether I was holding them to the general standard of that department. I like the system we have here in the department at USNA; whenever grades are assigned, a printout is posted of all the sections taught by the department along with the number of each kind of grade given in that section. This allows each of us to see how we fit in and adjust our grading if we feel it is appropriate.

In talking to the other professors here, I heard a lot of good ideas about making up and grading exams. I'll finish by listing a few of these. One professor pointed out that one should always have a clear idea of what is being tested. For some instructors, that means straightforward problems and "no tricks." Others prefer to mix the straightforward problems with more challenging ones. (It seems to me that the difference between a "trick question" and a "challenging question" is often a matter of whether you are the student or the instructor.) One professor said that he felt that tests should be straightforward, and that the place to assess deeper understanding of mathematical concepts was in project assignments, where time pressure isn't so much of a factor. Several people talked about "grading positively," looking for students to show what they know rather than what they don't know. Patience with minor errors was mentioned by several instructors, and one of them pointed out that the errors are new to them even if they aren't new to us. One professor suggested that students be penalized more for unreasonable wrong answers (like baseballs that travel at 2000 miles per hour) than for reasonable ones, unless the student leaves a note stating that the answer is unreasonable but he/she doesn't know where the error is.

(Comments welcome:
stanford@nadn.navy.mil)