EDITOR'S NOTES

I'd like to thank all the contributors to this issue of *Mathematica Militaris*. Your efforts have made my first experience as Editor-in-Chief, enjoyable, and very rewarding. I expected to see a great deal of professionalism and commitment on the part of faculty members at the service academies. However, even these high expectations were surpassed. I'm am confident you will find this collection of papers as interesting and thought provoking as I did.

As the new Editor-in-Chief, I would like to give a special thanks to my predecessor, LTC Phil Beaver who has recently left West Point for a job at the Pentagon. His excellent contributions as the Editor-in-Chief helped to continue to improve the quality of this journal. We all wish the very best for LTC Beaver, and hope to keep in touch.

In an effort to continually improve *Mathematica Militaris*, and leverage the capabilities the Internet has provided, we plan to establish a web page in the near future that will allow users to peruse all the previous issues of *Mathematica Militaris*, as well as search for key words, authors, titles, etc. We will however, continue to produce a hardcopy of the publication just as we have done in the past. We will advertise the web address as soon as the site is established.

Enjoy the articles, and keep your ear to the ground, for soon we'll be soliciting ideas for the theme of our spring issue, and the subsequent call for papers.

Best wishes from West Point.
Mike Johnson

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SUBSCRIPTIONS TO *MATHEMATICA MILITARIS*:

If you would like to be on our mailing list, please send your name, address, and affiliation to:

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Overview

We have gathered a large group of tremendously interesting articles focusing on "Building Bridges and Breaking Down Barriers". This theme was interpreted quite differently by many of the contributing authors. However, this makes for a quite interesting collection of ideas.

We begin with an article by Lt Jim Patrey and Dr. Kieth Carlson from the Department of Behavioral Sciences and Leadership, USAFA. This paper, rich in historical facts, shows how deep and mutual the relations between Psychology and Mathematics/Statistics have been throughout the history. Next, Dr. Deborah Arangno reveals all the stages that a "pure mathematician" went through, until finally realized that, "theory and application are complementary tools in our effort to understand the complexity and diversity of the real world", and that "…The scientists who fail to recognize, or refuse to admit this, call to mind so many blind men groping the proverbial elephant in vain labor to divine its nature."

Next, Major Robert Gilchrist, from the USAFA expresses his opinion that our graduates need diversity and cautions against over specialization. He challenges us, the instructors, to ensure we are providing the *best* education for our students in this regard through our activity in the classroom and our influence with curriculum development. He provides several suggestions to break down barriers between departments as well as building bridges between theory and application.

The next four articles present successful projects and ideas implemented in the classroom. First, Major Gerald Kobylski describes the Interdisciplinary Lively Application (ILAP) program at the USMA. He also goes into the details of his first hand experience of developing and implementing an ILAP in the core math program at West Point. Dr. Jim Rolf was very successful implementing a multidisciplinary project in his Numerical Analysis class at the USAFA. Here he was able to bridge the gap between theory and application for his students. Additionally, in order to effectively create the mathematical model, his students were exposed to several other disciplines outside of mathematics. Yet another success story is described in the article provided by Dr. Peter Turner, Professor in the Mathematics Department and Chair of the Executive Committee for Computational Science and Engineering at the USNA. Dr. Turner recounts the collaborative efforts of eight faculty members from six different departments, which are clear attempts at building bridges and breaking down barriers among disciplines. Dr. Amy Shell at the USMA wrote the fourth article that centers on activities in the classroom. Dr. Shell and several of her colleagues implemented a program where the Firsties (seniors) actually prepared for and taught calculus to the underclassmen. She provides the details as well as feedback from the participants who clearly felt it was a positive experience.

Major (Chaplain) Carlos Huerta provided the final article for this issue of Mathematica Militaris. Although the title of his paper is "Counterpoint", he doesn't exactly dispute any of the ideas presented by the previous authors. Instead he cautions that we cannot focus so much on building new bridges and interdisciplinary activities that we forsake our duties to teach the fundamental skills of mathematics. His article, full of historical references, emphasizes that our math majors must have a solid foundation in pure mathematics. With regard to interdisciplinary projects he states, “Surely studying these subjects can be a part of doing mathematics, but they are only one small tool in the arsenal of a good mathematician.”

I am confident you will enjoy reading each of the articles in this issue of Mathematica Militaris. They are diverse in their topics, rich in content and provide great insight to the wonderful activities and projects that occur in and around the math departments at our service academies.

The Role of Mathematics in the Development of Psychology

Lt Jim Patrey and Dr. Kieth Carlson
DFBL, United States Air Force Academy

The roots of psychology are traditionally portrayed as steeped deeply in philosophy and physiology. A strong argument can be made that mathematics had at least as significant of a role in the development in psychology and serves as an even stronger influence in modern psychology. Psychology's development as a science was presumably impossible without mathematical tools for conducting statistical analysis, which serves as the backbone of modern psychology.

Efforts to descriptively represent behavioral data have been documented as early as the 16th century. In 1532, Oliver Cromwell mandated the use of regular surveys on the causes of death known as "Bills of Mortality" in England. These data summaries presented diagnostic information and influenced a variety of economic and social policies. Michael Graunt, in 1662 (about the time Isaac Newton was working on physical laws of the universe), organized and extended the demographic data collected from these bills, and provided descriptive analysis that afforded estimation of life expectancies, birth/death and sex ratios, and percentage of men able to bear arms. Graunt significantly elevated the study of human behavior to a new level of sophistication through the use of mathematics. Rarely is any research conducted today without gathering basic demographic data. Much work in the field of health psychology and sociology today, places significant emphasis on precisely the same type of data and reporting that Graunt used over three centuries ago.

The decades that followed included the development of game theory. Laplace, Gauss, and others realized that quantification of the likelihood of future events was possible. As early as 1713, it was noted that game theory could be applied to phenomena other than games of chance; Bernoulli noted that this same probabilistic reasoning could be extended to "civil, moral, and economic problems." Condorcet conducted the eventual direct application to psychological phenomena in 1785 as he applied the probability theory derived from game theory to jury verdicts. In 1830, Poisson extended this beyond simple probability based theorizing, and used actual data on jury verdicts to assess what level of majority produced the best jury decision-making. This represents the application of mathematical principles to a qualitatively different type of human behavior - human decision-making.

The introduction of probability theory into the sciences culminated in Poisson's supposition of the “law of large numbers.” This realization of the tendency of distributions to form bell-shaped curves with scores clustering around the center, led to Quetelet's extension of the “law of large numbers” beyond games of chance to human behavior. Quetelet believed that human behavior could be powerfully represented in a concept of the "l'homme moyen" or "average man." This was a significant theoretical evolution - understanding phenomena by virtue of understanding central tendencies vice precise formulaic models. Spencer, an early sociologist, perhaps best stated the significance of this for the behavioral sciences: "if there is some precision, there is some science." The ability to explain general tendencies instead of absolutes (a pervasive scientific philosophy of establishing ‘laws’ was well-entrenched as the scientific doctrine at the time) enabled human sciences to overcome the diversity inherent in human behavior (i.e., it permitted psychology to separate *systematic* from *random* variance).

This concept of centrality gave credibility to the ongoing work of Weber on “just noticeable differences” in 1834. Weber's “just noticeable differences” formed the basis of a new field called “psychophysics” that focused on describing the relationship between the physical properties of stimuli and their internal perceptual representation via a mathematical formula (in fact, the foundation of psychophysics is sometimes attributed to Herbart’s “mathematical psychology”). Fechner extended Weber's work and defined a logarithmic law relating sensation and perception (known as Weber's law), foreshadowing the next significant mathematical breakthrough in the behavioral sciences. He noted that there were a great variety of factors that would produce deviations from the devised sensory-perceptual relative curves. His realization that this variability could be eliminated via methodological design was another cornerstone of the scientific evolution of psychology.

While Fechner and others (such as Quetelet) recognized that this variation was a component of a distribution, they generally considered it something to be eliminated or ignored. Galton, however, felt that the *atypical* vice the *average* held significant value. Galton was interested in intelligence; in particular, he was interested in explaining exceptional intelligence as genetic in origin. He recognized the importance of centrality in characterizing a distribution, but he realized that the variation of a distribution also was of considerable value. Galton’s focus led to his conceptualizing deviations from centrality as important information and ultimately to his conceptualization of an actual numerical representation of the relationship between variables, which he named “correlation,” in the late 19^{th} century.

The “discovery” of a test statistic to quantify relationships between variables presaged a significant transition in the conduct of psychological research. Pearson followed Galton’s work with his formula for deriving the correlation (commonly known as Pearson’s correlation and arguably the most commonly used test statistic today), Gosset with the t-test, and Fisher with analysis of variance; each of these test statistics, derived from issues in psychological research, enabled scientists studying human behavior to approach research from a new perspective and with new, powerful tools. In fact, it led Pearson to throw down the scientific gauntlet with a single phrase – “statistics on the table.” His point was that the objective means of settling scientific disputes was available and that a scientist was required to provide data if they wanted to be involved in any of these professional discussions. This became a much-publicized scientific debate affecting social policies in the early 20th century and pervades the current philosophical stance of most scientific disciplines including, and perhaps especially, psychology.

A great acceleration in the individual growth of psychological and statistical sciences ensued as Pearson's empirical philosophy gained preeminence. Multivariate statistical analysis such as path analysis, factor analytical techniques, nonparametric statistics, and other more complex regression and variance analysis, have burgeoned and enjoyed great use by behavioral scientists as the interaction and interdependency of these two disciplines continued. And where are we today? While the subtleties of missing data estimation, survival analysis, and handling Poisson distributions still challenge psychologists and mathematicians, I'll argue that there are two great collective challenges for our disciplines right now – cognitive modeling and complex systems (though there is admittedly some overlap between these domains). Cognitive modeling is a sub field of artificial intelligence that focuses on building mathematical models of human cognitive processes, generally using software. The challenge is twofold (as have been most that have been discussed thus far) – psychologists must discern enough about human cognition to quantify it precisely, and mathematics must be sophisticated and flexible enough to accurately represent this quantification. The more traditional linear statistical methods cannot satisfy the data analysis requirements imposed by approaching human behavior as a dynamic, complex system. Therefore, as our understanding of human behavior as a complex system acquires greater complexity, we are beginning to rely on nonlinear analytical techniques with greater regularity.

Psychology has much cause to thank mathematics – it has enabled it's evolution as a discipline, by providing the means to quantify, describe, and contrast human behavior, refine our methodology, and enable a high level of precision and sophistication in our understanding of psychology. Similarly, psychology has provided a host of real-world problems for mathematics and given a great push to the field of statistics. We expect to see this synergy to continue – psychologists will keep digging up new challenges for mathematics and mathematics will keep providing new tools to address these challenges!

The Blind Men and the Elephant

Dr. Deborah C. Arangno
United States Naval Academy

I used to be an elitist. There, I've come clean! I spent the greater part of my education in Mathematics as an intellectual snob, identifying myself as a "pure Mathematician", not to be confused with the "applied" sort. The disdain cultivated in me was also towards the laboratory-based applied sciences, of any area of study, and I particularly looked down my academic nose upon the Engineering disciplines.

This attitude planted me squarely at odds with my earlier and more formative training in the classical tradition, according to which the search for the Truth is the same quest, whether one embarks on the path of empirical science, Mathematical theory, or philosophical inquiry. Yet, I was convinced in later years that the theorist was the truth-seeker more inclined toward enlightenment, and that the Truth preferred to reveal itself in the study of pure form rather than in computation, error analysis, or random samplings. I was naturally inclined to believe that the Truth was deterministic, that real processes can be articulated by axiomatic systems, that problems ought to be reduced to canonical form, and solutions rendered as exact.

How very differently, I discovered, the engineers themselves viewed things. My first blunt exposure to this diametrically opposed perspective, was in the halls of a renowned technical school, where I found myself pursuing doctoral studies in Mathematical Physics. I learned quickly that the pure beauty of the formulation of a problem was not nearly as valued as the practical solution to that problem, even if such a solution was derived by brute force and trial-and-error. The celebrities here were the researchers who pioneered new stereo components (such as the equalizer, and "surround sound"), and who revolutionized acoustical systems and designed musical instruments with improved waveform capacities. They were the ones who explored alternative fuels, designed new "space age" materials for higher temperature tolerance and better performance, designed new methods for reliability testing, and generally labored to improve every day life, even if (I had once felt) they did so in the dark, stumbling from one discovery to the next. Ironically enough, it was they who had a very poor opinion of theorists, like myself. To them, theorists were armchair quarterbacks, full of erudition and producing little of concrete value or relevance. Their attitude was that the engineer and the practical scientist had "real" work to accomplish. After all, General Relativity was inexorably at odds with Quantum Mechanics, the latter at least could be verified in the lab.

Later in my career, when I was working on Space Defense at Norad/Space Command, and subsequently on "Star Wars" with the Strategic Defense Initiative, it finally dawned on me that any conflict between pure Math and applied science was unnatural and counterproductive. In our labors, we could not afford to distinguish between the "practical" and the "theoretical" in matters of national defense. Ironically, I found a new appreciation for "pure Mathematics" and discovered how the formulation of esoteric mathematical models can be implemented in the real world with dramatic impact. Improvising new systems, methodologies, paradigms, architectures, functionalities and unforeseen technologies, gave the process of abstract inquiry a concrete purpose. It made it meaningful in a tangible fashion, not merely for the beauty of the abstraction alone.

From battle managers to weapon-target assignments, from ballistic missile defense architectures to next generation weapons systems design, from basing strategies to verification and validation testing, from trade studies to feasibility analysis, what I found to be indispensable, it was the concerted collaboration among all of us; the various engineers, systems experts, computer scientists, operations researchers, and yes, even "pure mathematicians"! We could not have afforded to distinguish between the practical and theoretical, which would have only served to bind the scientist's right hand and the Mathematician's left.

What this means is that the mathematician cannot haughtily dismiss the labors of the laboratory researcher, and the engineer cannot regard the theorist as an egghead. Simply said, there is a natural conjugal relationship between theory and application, which we may – and must – use to our mutual benefit, in advancing our common labors.

However, our biggest challenge is not in convincing ourselves of this, but in instilling this appreciation in our students, and especially in the Service Academies. We ought to be genuinely concerned about equipping our students with both: the skills of abstraction, so they can envision possibilities otherwise unknown; and the methods of technology, so they can apply the principles of Mathematics and pure science to the development of those visions. Our concern must be how much rigor and generality we must sacrifice to appease the students' own demands for relevance and utility. We must be careful in crafting a response to the students' perennial and ubiquitous lament "Why do I have to learn this? When will I ever use that?!". We must make sure that their entire learning process is not reduced to the buttons on a calculator, or to the knee-jerk response to immediate and current demands of a job at hand. Too often, the engineering student uses a formula, which is not suitable to his application and entirely out of the context for which it is intended. This happens because the student disregards how the formula was derived, and does not appreciate the conditions under which the formula was expressed.

In this increasingly pragmatic and utilitarian ethos of modern culture, it is critical for us to assert the importance of abstract forms. We must remind the students of the validity – if not the priority – of universals. We can illustrate for them how Mathematics has always given us insight into reality of things, even those that elude us empirically, from imaginary numbers to "black holes" and anti-matter. It is important to emphasize that even when we lack the faculty to observe things, we can know their existence simply because they *ought* to exist, Mathematically! The scientists who fail to recognize, or refuse to admit this, call to mind so many blind men groping the proverbial elephant in vain labor to divine its nature. Indeed, any modern theories, from physical laws to those governing psychology and economics, which attempt to explain any facet of reality purely within the confines of empirical science, have proven to be inadequate. The implicit lesson of their dismal failure is that no reality can be explained, let alone understood, unless all dimensions of the subject under consideration are scrutinized with equal ardor. Surely, theory and application are complementary tools in our effort to understand the complexity and diversity of the real world.

Does Specialization Really Do Education Justice?

MAJ Robert N. Gilchrist
United States Air Force Academy

The world, and the technology that drives it, is changing at an amazing pace and it is clear now more than ever that the distinction among different technical disciplines within the mathematical sciences is becoming blurred. Nevertheless, specialization is still the focus of much of our academic training and our motivation for creating more specialized majors within our departments. Furthermore, unless one is seeking entrance to medical or law school, the generalist is looked down upon with some level of disdain. Take for example the "basic sciences" or "general engineering" majors at USAFA. These majors are looked upon as "fall back" majors for those who cannot "cut it" in more specific and, commonly considered, more difficult majors. However, there was a time when the Air Force Academy only offered one broad based major in which all cadets "majored." The remnants of this 50's era "major" still exist in our core curriculum today.

It is my belief that our focus on specialization originates in our own academic experience, which may not necessarily be suited for today's rapidly changing technological environment, and in our discomfort with teaching outside our areas of academic training. Furthermore, by taking a heavily major-focused approach to education, we are doing a disservice to our young officer candidates in requiring specialization at such an early stage in their education. Perhaps we should be encouraging a broader educational experience for our cadets, leaving them with a solid educational foundation from which they can specialize to meet the needs of the armed forces or to benefit their own personal growth.

I freely admit that broadening a cadet's educational experience might be done at the expense of expertise, but is expertise ever earned through education? My guess is that expertise is earned through experience with additional specialized education added at the appropriate time. After all, isn't our ultimate goal to produce general officers for the armed forces? These are people who have a broad base of knowledge and experience that allows them to take over and run any operation. When I consulted Webster's Dictionary I found the definition of "general" to mean "not limited in scope, area, or application." These are the type of personalities we treasure, so why not start now to cultivate them in the right way?

The question then becomes, "How can we maximize our efforts to produce better officers while maximizing the student's experience with mathematical sciences?" In addressing this question I believe it is critical that we be ready and able to take material from the theoretical to the applied rather than teaching the "beauty" of mathematics for its own sake. The theoretically inclined student will like mathematics for its own sake anyway, but the student who needs a more concrete example or application will greatly benefit from such exposure. I will assert without any supporting data that this type of student, the one needing the concrete, "real world" examples, represents the majority at the service academies. I propose the following taxonomy for applying mathematical concepts and for connecting applications to their origins in mathematics.

The "forward reference" is the most important reference an instructor can make in a core course. This is a chance for the instructor to show how mathematical concepts will be used in applications and in subsequent "subject" courses that the student will encounter. While reference to future use may not be immediately understood or appreciated by the student, making such a reference plants the seed of understanding in the student's mind. How many times have you had a revelation about a concept that you studied years ago but never really understood? In my experience, planting a forward reference speeds up this process immensely. A forward reference is especially important in facilitating a "backward reference," a reference by an instructor teaching an upper level applications course, to a mathematical or scientific concept taught at an earlier stage in the student's educational experience. Backward references close the loop, tying application to theory where, hopefully, theory was presented in an applied context at an earlier time. The table shows a small sample of forward and backward references that can be made among mathematical and scientific theories and their applications.

Topic Presented	Reference Type	Related Topic
Taylor series expansion	Forward	Small angle approximations for pendulums
Geometric series	Forward	Infinite/finite capacity queuing models
Expected value calculations	Forward	Decision analysis
Vector cross products and dot products	Forward	Static equilibrium in engineering mechanics or work concepts in physics
Equilibrium/steady-state concepts in Markov chains	Backward	Dynamic equilibrium in chemistry
Macroeconomic multiplier effect	Backward	Geometric series
Continuous compounding	Backward	Exponential functions

So, what can we do to build bridges among disciplines and increase our ability to make these forward and backward references? The answer is to broaden our own academic and pedagogical experiences. This can be done in several ways. One way is to teach courses beyond our "comfort zone," but within our general sphere of knowledge. Have the pure mathematician teach applied statistics; have the pure statistician teach engineering mathematics; have the operations research instructor teach real analysis. Teaching beyond our comfort zone does several things. It enhances department-wide faculty development, it fends off the complacency in teaching the same types of courses over and over again, and it reintroduces the excitement of learning to the instructor.

Another way to build bridges among disciplines and increase our ability to make forward and backward references is to teach in a completely different department. For example, the USAFA Department of Mathematical Sciences (DFMS) currently has an exchange program with the Department of Engineering Mechanics (DFEM) where each department sends instructors to teach in the other. This has two distinct benefits. Clearly, it increases an instructor's understanding of the relationship of mathematical topics to other subjects and, as an added benefit, it allows instructors to see how other departments operate from an organizational and administrative perspective. What department couldn't benefit from a new idea garnered from another?

Ultimately, departments with common interests should collocate in order to remove artificially imposed geographical barriers to communication. Some effort should be made to teach interdisciplinary or capstone courses that integrate several mathematical, scientific, economic, or financial concepts allowing the student and instructor alike to benefit from cross-pollination of ideas.

Frankly, all of these ideas I have presented take a great deal of time and effort to implement and it would be easiest to do nothing and conduct business as usual. But, if we are to break down the barriers among our disciplines and build bridges between theory and application, we must create a unified vision for what we want our young leaders to know and what the armed forces needs. I submit that this vision is a broad educational background based in application. The only way for this to happen is for the instructors to blaze that path.

And the Walls Began to Crumble

MAJ Gerald C. Kobylski
United States Military Academy

How can we as instructors better coordinate our efforts to maximize the students' understanding of mathematics and its applications? I believe an important and an effective way for us to do this is through collaboration with other departments. On the surface this sounds very simple, but those with experience in education know that this is far from easy. So, what can we do to facilitate collaboration? In this paper I will discuss several possibilities that include developing Interdisciplinary Lively Applications (ILAP), interdisciplinary homework problems, one on one discussion between the directors for the courses, and finally, meetings between departments. These ideas are not new to multi-disciplinary education. My goal here is to discuss how we conducted these and the successes we observed. My observations come primarily from a multivariable calculus course in the Department of Mathematical Sciences, United States Military Academy.

There are many ways of breaking through the barriers that students have amongst disciplines, the most effective of which I believe is an ILAP. When ILAPs relate concepts from different disciplines to students, they create a much deeper understanding of concepts taught. I have found that ILAPs are most effective in breaking down student compartmentalization when the students are presently studying the topics included in the ILAP. The reason for this is that the course material is still fresh in their minds. This finding motivated me to choose economics as my partner discipline for my first ILAP; all of our students took an economics course in the same year we were teaching them multivariable calculus. In developing this project, I included ideas that the students encountered in the economics course such that the project's requirements tied together the two courses. Some of the economics topics included were the law of diminishing returns, marginal cost, marginal revenue, and maximum profit. A full discussion of this problem can be found in "Making Money with Mathematics," *Mathematica Militaris*, Volume 9, Issue 1, Spring 1999.

The development of the ILAP took a lot of time, but it definitely had its benefits. The hard work the students put into solving the economics problem proved successful in deepening their understanding of mathematics and gave them an appreciation of its power to solve real world problems. The project was also successful in furthering the students' understanding of the concepts they were learning in economics. Some of the concepts and formulas encountered in economics made more sense when their mathematical development was explored. For example, students were usually quick to memorize in economics that a company maximizes profit when marginal revenue equals marginal cost, or when $MR = MC$. However, not all students understood the basis of this relationship. In our calculus courses, they had repeatedly maximized functions by taking the derivative and setting it equal to zero. Knowing that total profit was equal to total revenue minus total cost, or $profit = TR - TC$, many students discovered in our ILAP that taking the derivative of this profit function (with respect to quantity produced) and setting it equal to zero yielded $MR - MC = 0$, or $MR = MC$. With this single requirement of the ILAP we dented the wall between economics and mathematics. The students were not the only benefactors of this ILAP; the instructors also learned a tremendous amount about the mathematical foundations for several major concepts in economics.

In order to achieve some of the benefits described, you do not have to rely on a formal ILAP, which as mentioned could be quite time consuming. Instead, you could develop problem sets that explore concepts in both courses. This approach is much easier because you do not have to develop a complex scenario tying together all of your requirements. During a different semester of multivariable calculus from the one mentioned above, a goal was to emphasize the concept of vectors in our course and in the core physics course. Previously, the two departments had used somewhat different terminology and approaches in teaching vectors. As a result, students believed they were learning different concepts. We incorporated a few straightforward problems from the physics course concerning motion into our syllabus. Many students realized that just because you were in a different department, it did not mean the theory was different. In retrospect, I wish that I had utilized even more problems that related various concepts in both courses.

Another great way to break down the barriers between courses is for the directors of the courses to meet as frequently as possible throughout the course. As the course director for a multivariable calculus course, I met with my counterpart from the physics course that my students were taking in order to determine what alignment of teaching was possible. We decided to try to make connections between the two courses in vectors, motion, and differentials. We added the differentials topic to the course at his request because the students in his course traditionally had a hard time with uncertainty analysis. This was not difficult to do since differentials was simply an application of the concepts we were teaching. With the added emphasis in these areas, the students seemed to make the connection between the two courses much more easily. The big hit was differentials. The physics course director raved about how much better his students understood the use of differentials in calculating uncertainties in measurements during their labs.

Using some of the same teaching aids that were used in physics also helped in breaking down the barriers between physics and mathematics. When comparing our courses prior to the beginning of the semester, we found that the physics instructors had some aids they used when teaching vectors. We had them constructed for our course. Seeing the same teaching aids in both courses, although very simple in nature, I believe helped in making the concept of vectors seem like it was the same in both courses.

Exchanging instructors for a semester can also help break down barriers between departments. This may be difficult for a department to do but its results are well worth the price paid. As an exchange instructor in the Department of Systems Engineering, I learned a different approach to teaching several concepts and was able to share these approaches in the Department of Mathematical Sciences and was also able to share some of our ideas from the Math Department with Systems.

In the modeling course I taught there, I discovered numerous mathematical applications and was able to draw these out in our class discussion; this took little effort because most of these students had taken multivariable calculus in the previous semester.

The final avenue I will discuss that can open the lines of communications between departments is perhaps the most difficult, but may have the most potential. The idea is to have interdepartmental meetings. These meetings are usually most effective when only two departments are involved and can occur as little as once a semester. All of the instructors in each department can attend or just those in applicable courses that you may be trying to correlate. Last year we held what I believe was our first interdepartmental meeting with the Department of Systems Engineering. During the hour-long meeting, instructors expressed various teaching philosophies. A good question to open up for discussion as such a meeting is: what can / should each department do more or less of? Those who attended this meeting came away with a better understanding of the teaching approaches of each department, a positive result for a first meeting. Perhaps most importantly, we met some of the people who were teaching similar concepts as we were teaching. Meeting the right people is sometimes the hardest step in interdepartmental communication.

Certainly it is not always feasible for entire departments to gather together. An option might be to invite guest speakers from other departments. One day our course hosted several instructors from the Department of English. We asked them to discuss how they taught the students to write essays and what their expectations of the students were. The reason for our interest was because we had our students do a significant amount of writing in their projects and we wanted to ensure that we were holding appropriate expectations for their writing. This short meeting was successful in that our instructors felt a little more comfortable with the writing standards for our students.

Collaboration between instructors from various disciplines can be a wonderful education tool. But as powerful as collaboration can be, there exist many obstacles that can prevent effective collaboration. The most common obstacle is instructor time. As educators we are so busy teaching our students, grading exams, developing lesson plans, and conducting research that it is hard for us to find the time to interact with other departments. The best way to overcome this obstacle is to get as many people as possible involved in order to share the workload. Several instructors working together can easily develop effective ILAPs or problem sets in a short amount of time. At a recent Mathematical Association of America workshop on the development of ILAPs held at Western New England College, several teams consisting of four or more individuals developed very interesting interdisciplinary projects in just over a day. The obstacle of time becomes even more insurmountable when we do not know who our peers are in other departments; communicating with other departments then becomes even more difficult. Interdepartmental meetings can easily solve this problem.

Another barrier between collaboration might be the lack of interest by instructors in different disciplines. Some engineering and mathematics instructors may have little interest in the humanities. Likewise, instructors in the humanities may have little interest in engineering and mathematics courses. I must admit that when I first began teaching, I did not have much interest in physics. Yet the basics physics course that all of my students took probably had the most applications of the calculus I was teaching. I think lack of knowledge fosters lack of interest. When I discovered the applications of mathematics in physics I became very much interested in the subject. When the instructors from the Department of English shared their ideas with us, I also became very interested in what and how my students were learning in their English courses. I believe the instructors from the Departments of Physics and English came away with similar feelings.

A final barrier that I will mention is the organization of each department, something which is school dependent. Some courses have one person responsible while others have several running their own sections. A centralized system makes it easier to coordinate programs with especially when courses have a large enrollment and many instructors. At USMA we are fortunate in that most of our courses have a single director so that it is very easy and efficient to share ideas about courses. In institutions that are decentralized, collaboration with one individual may result in changes in that individual's section of students but in none of the others. Thus, some may feel the collaboration is not worth it. In this case all of the instructors involved in the interdisciplinary work should share their product with others. I have yet to meet someone who would not take something for free, especially a neat interdisciplinary problem!

In summary, collaboration between disciplines has compelling results. I have discussed in this paper several possibilities that enhance collaboration. These include developing Interdisciplinary Lively Applications (ILAPs), interdisciplinary homework problems, one on one discussion between the directors for the courses, and finally meetings between departments. There are several barriers that can prevent such collaboration with other departments. Like our students, we instructors can become very compartmentalized by thinking that our teaching approach in our courses is the only possible way. With a little effort, these barriers can easily be overcome. Although in the past few years educators in mathematics have made significant strides regarding interdisciplinary work, there are still numerous walls between departments existing in the students' minds that need to be broken.

Ship Heave Effects During Deep Sea Drilling: Development of an Active Heave Compensation System

Dr. Jim S. Rolf
United States Air Force Academy

Introduction

In the spring 2000 semester, I asked my Numerical Analysis class to work on a project involving the mathematics of modeling the heave motion of the of the ship *JOIDES Resolution*. All modeling was to be based on various approximation and interpolation techniques[1] using data collected from the 188th scientific party between January and March of 2000.

The intent of this project was to give the students experience with real-world

[1] These techniques included interpolating polynomials, least squares techniques, and interpolating cubic splines.

data, to improve their modeling skills, and to do these in a context that might prepare them for experiences beyond their insulated life here at USAFA.[2]

The Problem

The statement of the problem given to the students follows.

"Deep sea drilling for scientific purposes is used to explore new energy sources, understand past climactic conditions, predict future climactic phenomena, study tectonic plate movement, and study volcanism. All of these scientific explorations require retrieving core samples from the ocean floor. Thus, drilling is an essential tool of the marine Earth science community in the U.S.[3] and compelling scientific objectives require ocean drilling as a means of acquiring core samples.[4]

"Good" core samples are imperative for meaningful data. Each core sample is 10m long by 2 inches in diameter. Ideally, 100% intact core samples will be obtained. In practice, however, on average only 50% intact core sample is recovered during drilling operations. Scientists would like to increase this percentage. A primary requirement for obtaining good core samples is maintaining a constant weight on the drill bit on the ocean floor. This down hole weight-on-bit (DWOB) is affected by the heave of the ship due to ocean swells during drilling. Consequently, effective heave compensators are important to mitigate changes in down hole weight-on-bit.[5]

Marine drilling systems have certain elements in common. A drilling system has a method of rotating the drill string (usually connected lengths of pipe), either with a rotary table/Kelly drive or with a top drive rotary. A top drive system is greatly preferred, because it allows additional joints of drill pipe to be made up onto the top of the drill string without raising the bit any appreciable distance off the bottom of the hole.[6]

In any floating drilling operation, ship heave pulls weight off the bit as the floating rig rides to the crest of a wave and places weight back on the bit as the rig rests in the trough. Consequently, drillships must compensate for ship heave on the drill string. Otherwise, the bit will bounce on the bottom of the hole making drilling virtually impossible. Heave compensators generally consist either of a piston-and-cylinder assembly that is crown mounted at the top of the derrick or mounted directly under the traveling block, or of a motion compensated drawworks. Both types of motion-compensation systems work to maintain relative position between the top drive and the bottom of the well.[7]

Many drillships use an active heave compensator. An active compensator reacts differently than a passive drive in that the system attempts to anticipate up-and-down movements instead of reacting passively. The system measures real-time rig displacement by integrating signals derived from an accelerometer and inclinometers. From this continuous calculation, the system adds or subtracts the residual motion imparted to the drill string by raising or lowering the drill string

[2] United States Air Force Academy.
[3] [1], p. 1.
[4] [1], p. 1.
[5] [3].
[6] [3], p. 32.
[7] [1], p. 32.

relative to the vessel. The work done by the system removes up to 95% of the unwanted excitation imparted to the drill string by the passive compensation system. In other words, an active system greatly reduces the magnitude of the variation in drill bit weight on the bottom of the borehole, which for a passive system can amount to as much as 20% of the weight of the drill string. [8]

Pressing needs for deep sea drilling include evaluating the effectiveness of the current heave compensation systems and minimizing the effects of ship heave on core recovery.[9]

Your task is to recommend a software solution that will act as an active heave compensation system, anticipating the up and down heave of the drillship.

In order to design this software, you will be given heave data from the Antarctic leg of the 188th scientific party of the *JOIDES Resolution*. This data was collected between January and March of 2000. Your code should model this data and form the basis of your active heave compensator."

Student Reaction

I placed each student in a team of three in order to give them meaningful experience of working in teams that would better simulate the context in which they will likely solve problems in the future. I asked each team to turn in one final product that included an electronic copy of all software and a written document with any necessary charts, graphs, hardcopy of all software code, and recommendations for future work that could improve the predictive ability of approximation models used to model the ship heave motion.

Student reactions to this project were very positive to both the interdisciplinary nature of the problem and the team context of modeling the given data. Student comments included: "This is a useful topic. I can really see how it applies." "I think this stuff is really valuable." "It was great to see real life examples and uses for class materials." More than one student saw connections with things being studied in other engineering courses.

My Reflections

This genesis of this project was in research conducted with geologist Greg Myers of Lamont-Doherty Earth Observatory of Columbia University. And this working relationship happened to be one that was initiated out of a personal friendship. Consequently, I'm not sure that this kind of experience can be replicated in a reliable manner to increase participation in interdisciplinary issues (unless one has lots of personal friendships with scientists in other fields!)

However, I was able to take a piece of this research and make it accessible for the students to gain valuable experience modeling the data. I was particularly impressed with their thoughtful suggestions about how to improve the model in future work by using periodic basis functions (something we had not discussed in class).

I found this project a valuable interdisciplinary experience for my students. They gained valuable experience with real-world data in a way that contextualized the learning experience. This experience reinforced both the

[8] [1], p. 33
[9] [2], p. 26.

mathematical concepts as well as heightened the modeling experience.

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[1] The Non-Riser Drilling Vessel for the Integrated Ocean Drilling Program: A Report from the Conceptual Design Committee. Performance Specifications, on board scientific measurement capabilities and survey of drilling vessels. A Report to the U.S. National Science Foundation. March 2000

[2] Ship heave effects while drilling: observations from Legs 185 & 188. D. Goldberg, G. Myers, G. Guerin, D. Schroeder and the Legs 185 & 188 scientific parties. JOIDES Journal, Vol 26 No 2-2000. pp. 26-29.

[3] Personal conversation. Greg Myers, Manager of Technical Services Lamont Doherty Earth Observatory, Pallisades, NY. March 13th, 2001.

Center for Computational Science and Engineering

Dr. Peter R. Turner
United States Naval Academy

Background

This report provides a record of the activities concerning the formation and early development of the USNA Center for Computational Science and Engineering, CSE. The formation of such a Center was first proposed at a USNA Faculty Colloquium in October 1999. The idea was enthusiastically received by a significant number of faculty members from a variety of departments.

The Academic Dean and Provost, Dr William C. Miller, proposed in early 2000, the establishment of a program of USNA Teaching Fellowships in order to support major new teaching initiatives. It was natural to propose that one such Fellowship be awarded to follow up the energetic first few months of activity in CSE with a concerted effort to build on these beginnings. This proposal was rewarded with the first USNA Teaching Fellowship.

One of the first tasks completed in the development of our Center was the adoption of Vision and Mission Statements. These include a local definition of CSE. The vision statement consists of the first paragraph of the Mission Statement, which is included here to help the reader understand the context of what follows.

*The **Center for Computational Science and Engineering** (CSE) is a multidisciplinary organization promoting the use of computation in science and engineering with strongly positive implications for enhanced faculty research, midshipmen research and highly relevant undergraduate educational programs.*

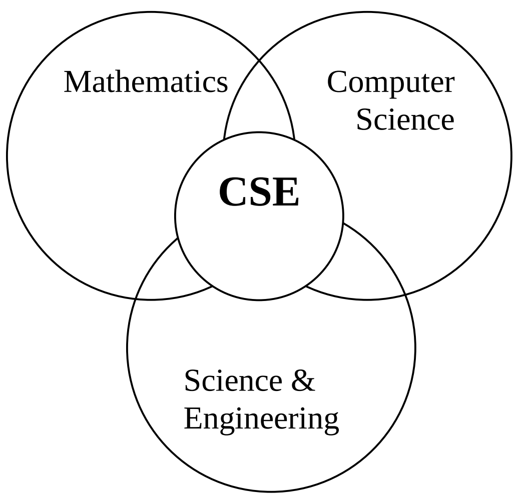
In broad terms Computational Science and Engineering involves using computers to study scientific and engineering problems, complementing the areas of theory and experimentation in traditional scientific investigation. CSE combines domain expertise with expertise in modeling and computational areas such as numerical analysis, algorithm development, visualization, and software implementation.

The Center for CSE is working to:

- Promote USNA research in CSE through improved collaboration and communication,
- Foster cooperation between USNA researchers in CSE and outside institutions, including Navy and other government laboratories,

- other colleges and universities, and private industry,
- Become a recognized Center of Excellence for CSE,
- Develop undergraduate multidisciplinary research and educational opportunities in CSE at USNA, and
- Assist CSE researchers at USNA in accessing and using high performance computing machinery.

In describing the discipline, or more accurately the multidiscipline of CSE, the following Venn diagram is often used.



Here we see CSE as larger than just the intersection of the three circles but still living inside their union. I believe this may be the correct model. It shows CSE as having separate intersections with mathematics, computer science, and science and engineering which is a reflection of reality. There are certainly scientific discoveries that have grown out of computational science – much of the human genome project for example. There are impacts of CSE on mathematics in the development of new algorithms and theories. There are clear computer science links through visualization and data mining, for example, where CSE has impacted computer science rather than just borrowing from it.

Some advocates of CSE regard this diagram as being inverted in the sense that CSE could be seen as encompassing all the other disciplines. It is easy to fit certain components into the other intersections. For example the intersection of mathematics and computer science could describe the discipline of Computation Science. It could certainly include numerical analysis, scientific computing and visualization. The intersection of mathematics with science and engineering could include traditional disciplines such as mathematical physics as well as quantitative economics, digital signal processing, image analysis, etc. It is clear that some of these have significant computational aspects and so perhaps almost merge into CSE. Computer Engineering would be a natural candidate for the computer science/ engineering intersection.

CSE Activities at USNA

In the two years since the original proposal for the formation of our Center, CSE has had a significant impact. There have been curricular developments, which have involved team-teaching across discipline/departmental boundaries to students from several majors, and establishing of a Second Discipline in CSE for the new Information Technology major. We have begun a Colloquium Series with a very successful “kick-off” event last spring that attracted well over 100 participants from almost every department at the Academy. There are advanced Midshipmen Research projects in CSE within the Trident Scholar program this year and we expect further such projects in future years. There is multi-departmental support for an Advanced Computing Center, which would provide hardware infrastructure for CSE teaching and research activities, and there are proposals for multidisciplinary research projects in CSE. The remainder of this section describes briefly two courses that have been developed for our CSE Curriculum.

The CSE Curriculum

Introduction to Computational Science and Engineering

This new course was designed by a team of faculty and offered initially in the Spring semester of 2001. The course is being offered again in Fall 2001. It is designed to have four projects – each lasting about 4 weeks. Faculty from various departments proposed these projects, and some were team-taught. For the students, the projects were also team efforts, where teams of three were most commonly used. In one of the projects, a jig sawing approach to the student teams was adopted. The basic idea here is that the team-members were also assigned to one of three *expert teams* who had the responsibility for gaining a deeper understanding of one of the key aspects of the project, implementing that piece of the team project and educating the remaining team members in that area.

The faculty members most closely involved with the Introduction to CSE course were: **Peter Turner, Erik Bollt, Joe Skufca, Mike Chamberlain** and **Joel Modisette** (Mathematics); **Tom Zak, Matt Baker** and **Pam Schmitt** (Economics); **Kevin McIlhany** and **Heath Hanshaw** (Physics); **John Burkhardt** (Mechanical Engineering); **Bob Voigt** (Electrical Engineering); and **Chris Brown** (Computer Science).

There were eight initial project proposals, four of which are outlined below. The group as a whole was responsible for the final selection of the four projects used. All the projects were worthwhile and the final decision was made easier only by "convenient" scheduling problems, which reduced the choice for us. Eventually, the course was taught by Zak, McIlhany, Burhhardt, Turner, Bollt and Skufca, with valuable in-class assistance from Brown and Chamberlain. While this sounds expensive for a single section, all were happily volunteering their efforts and (for most class meetings) only one instructor was actually conducting the class.

Eighteen Midshipmen completed the course in the Spring and another eight are taking it this Fall. These students have come from four different majors. The course was being designed at the same time as it was first offered for registration and was (at that time) only accepted as a 400-level elective in two majors. The course was only listed as a mathematics (SM) elective and so was probably not even considered by engineering majors unless their advisors were directly involved.

The four projects selected for Spring 2001, were on:

Monte Carlo simulation for Financial Planning (Zak)

Most people have an interest in planning for the future. In particular people, often wonder how much money they must save to accommodate anticipated future expenses such as a child's schooling, or retirement. Traditional financial planning approaches plug in personal information, such as age, financial information, such as savings, and make assumptions about unknown future events, such as expected rates of return, to determine if one's saving plan is consistent with one's expected future expenditures. Slightly more sophisticated attempts vary the assumptions slightly to get "upper and lower limits." Both of these methods are flawed because they provide point estimates of what are inherently uncertain outcomes. Unknown future values are better treated as random variables. The Monte Carlo method runs thousands of chance scenarios that produce a distribution of outcomes that provides a good deal more information about the likelihood that one will actually achieve one's goals. The approach does not eliminate uncertainty, but can provide a probability that a particular financial strategy will accomplish its objective.

Tracing algorithms (McIlhany)

This interesting project gave students a chance to study some of the algorithms used for ray tracing in a variety of scientific settings. The central theme was on light rays and computer graphics packages. Students used both Vista Pro (a topographic scene rendering program) and Strata 3D (a graphics and animation generation package). The effects of textures on light reflections and the algorithms used to simulate these are discussed and the students then use the packages to create two "movies" – one a fly-through of a particular geographical location (real or fictitious) and the other an animation created from scratch. Attention to appropriate lighting effects was an important aspect.

Sea-floor characterization using acoustic backscatter (Burkhardt & Turner)

This project used random walks and Monte Carlo simulation to study a particular aspect of ray tracing – the scattering of sound waves off the sea floor – and how this can be used to characterize that sea floor. A simple backscatter model was developed and simulations produced for this. Multivariable minimization was then used to estimate the parameters in the model for test data. This project had three distinct aspects: modeling, simulation and parameter estimation. A jigsaw approach was used in which each project team included one member from each "expert team". The expert teams were responsible for one of the three parts of the project – and for educating their team members in that aspect of the overall process.

Efficient paddling of a kayak (Bollt & Skufca)

In this project, students develop a one-dimensional, physics based model of paddling a kayak using basic laws of motion. An appropriately simplified model can be understood by a student who has completed a first course in physics. The primary constraint that makes this problem interesting is that the length of the stroke is fixed. This constraint requires a numerically integrated solution to a system of differential equations with the added requirement of using a shooting technique to obtain appropriate resolution.

The objective is the determination of the "best" stroke. This is left as a very open-minded question. Determination of appropriate Measures of Effectiveness (MOEs) is fundamental to the overall project – for example, highest maximum speed, highest average speed, efficiency, furthest distance traveled in some time.

Schedule

The class schedule for the Spring 2001 semester allowed for 43 class meetings. Ten were allocated to each of the projects, with a general introductory session, a final class meeting to discuss the course, and a one-day hiatus between the second and third projects to allow for any snow days or a review of the course thus far. In fact there were no snow days, and the extra day was allocated to the students working on their ray tracing projects. A similar schedule was used in the Fall although the order of the projects was changed.

Dates	Project	Instructors
Mon Jan 8	General Introduction	Prof. Turner
Wed Jan 10 – Fri Feb 2	Monte Carlo Simulation for Financial Planning	Prof. Zak
Mon Feb 5 – Wed Feb 28	Tracing Algorithms	Prof. McIlhany
Mon Mar 5 – Mon Apr 2	Sea-floor characterization using acoustic backscatter	Profs. Burkhardt & Turner
Wed Apr 4 – Wed Apr 25	Efficient paddling of a kayak	Prof. Bollt & LCDR Skufca
Fri Apr 27	Review	All

Numerical and Statistical Analysis of Experimental Data

This course was a response to a perceived need. The course is *only* available to our Trident Scholars for whom it is required. It is being taught for the first time this Fall semester by Profs Gary Fowler and Peter Turner.

The need for this course has become apparent over recent years. An increasing number of Trident Scholar projects fall in the broad area of Computational Science and Engineering and many involve the generation and then analysis of experimental data. Although the students concerned are among our brightest, it is clear that they have never been given the opportunity, or perhaps realized the need, to study this important aspect of their work.

Examples in the recent past have included fitting very high degree polynomials to experimental data – *in order to estimate rates of change* of the measured variable. Another case included samples of four data points for different versions of a physical engineering component. For each sample of four, we were given the mean, standard deviation, and minimum and maximum values! It would have been at least as informative to give the original measurements, but whichever way the data was presented, some careful analysis would be needed to justify drawing any statistical conclusions. Areas such as model fitting and parameter estimation also occur frequently.

The course uses examples from recent and current Trident projects to first show the need for such a study and then gives pointers to what can or should be done. Some of the topics are: a brief discussion of errors and their propagation; what data to collect; numerical analysis of data (Curve fitting, Parameter estimation, Numerical differentiation and integration); statistical analysis (Sample size, Choice of distribution, Presentation of results). An important aspect of the course was the inclusion of brief student presentations on their work, where possible current problems facing midshipmen in the course were used to frame the discussion.

Although this course is being taught entirely from within the Mathematics Department, it really is a CSE course. The intention is to provide students of science and engineering with the tools they need and, most importantly, to make them *aware of the questions*. The students concerned are certainly pursuing *computational* science or engineering projects; and, the course is not in any sense a theoretical mathematics course.

Summary and further information

In this article I have attempted to give a brief view of the activities of our new center for CSE – which is clearly an attempt at building bridges and breaking down barriers among disciplines.

The formation of the Center was first proposed in October of 1999 and it already has two new courses, two Trident Scholars, a new Second Discipline program, a highly successful inaugural CSE Colloquium, and embryonic research projects to its name. The membership of the Center includes about 30 Faculty members from 9 different departments. It has certainly had success in at least piercing the defenses surrounding isolated departments. The bridges need reinforcing but at least they are there for those who tread carefully. Anyone who wants more information on our Center should contact the Peter Turner at prt@usna.edu or (410) 293 6732.

Firsties Teach Calc at USMA

Dr. Amy Shell
United States Military Academy

This fall the Department of Mathematical Sciences at USMA offered our Firstie (senior) math and operations research majors the chance to teach a class in our third core math course, MA205 Multivariable Calculus. This year the Academy started sectioning cadets in all core courses based on their company. Two to three companies are represented in each section. Theoretically, this allows the cadets to conveniently form study groups and for Company Tactical Officers (TACs) to visit classes. This resectioning sparked the idea to see if Firsties would like to teach a class. MA205 program director COL Joseph Myers spearheaded the idea, "I was telling my Mathematical Analysis II (MA487) class that I was going to have to miss a whole day of classes to sit on a board and mentioned to CDT Nick Clark that his TAC had visited my MA205 section. We joked about CDT Clark teaching the section while I was gone. It went from a funny idea to a neat idea. I remembered a proposal from the former department head, BG (Ret) Arney for a one credit Firstie seminar in Math Education. Letting all Firsties teach a class seemed like a small step in that direction. My 487 class and I talked about the possibilities and interest for every Firstie to teach a class."

The benefits of having an upperclassman teach a section is two-fold. First, it gives the Firsties the chance to experience teaching; the preparation required and the difficulty inherent in teaching 18 individuals with varied learning styles. As Col Myers put it, "it requires them not just to understand it their way, but to understand how others think and to help guide their understanding. It generates more of an appreciation for what it means to understand (with some depth) and communicate technical topics." The teaching experience also helps the Firsties decide whether or not they might want to return to USMA as a rotating faculty member.

Second, the Yearlings (sophomores) in the section get to see what is open to them as upperclassman. As COL Myers describes it, "Lower classmen already see Firsties as leaders in the military and physical programs. This is their chance to be seen as academic leaders as well. This is the Firsties' chance to share with them about learning and to help develop them academically. This gives Firsties the chance to share with the Yearlings in their company their perspectives on learning, wrestling with problems, and how they see core math connecting to the rest of the curriculum." This perspective is invaluable to the Yearlings.

Five Firsties took us up on our offer: CDT Nick Clark taught a lesson on the multi-variable chain rule, CDT Todd Hildebrant taught two lessons on LaGrange multipliers, CDT Dennis Mackin taught double integrals over general regions, CDT Jay McGee looked at the applications of the double integral to probability, and CDT James Starling taught the introductory lesson on multi-variable functions. Each Fristie was paired up with a MA205 instructor that had cadets from the Firstie's company in their section. The Firstie chose the topic they wanted to teach and prepared the lesson with general guidance from the instructor. In order to give the Firstie the full experience, and complete control in the classroom, he instructor was not in class the day of the lesson. In a few cases, the class was not informed ahead of time about the guest instructor. This caused some surprise and confusion, not only among the class, but with some faculty roaming the halls. But it is always good to shake up the Yearlings a bit.

The classes went well and the overall response was positive. The Firsties enjoyed preparing and presenting the lessons, while admitting that is didn't always go as they had expected. "The most prominent thing I learned from this experience" said CDT McGee, "is that teaching is much more difficult than it appears. It is much different teaching the material in front of the class than simply having a mastery of it. All in all, it was a rewarding experience. I mainly learned teaching is harder than it seems." CDT Mackin agreed, "Definitely an experience worth doing. This was a great opportunity to get a feel for what it is like to have 20 cadets staring at you with blank faces and confused looks."

Cadet Clark commented that the experience "made me realize how far I had come since MA205. Things that the class struggled with just seem like common sense now. It was also interesting to see how different students learn. In advanced mathematics, I think, most students tend to learn in a similar manner and at a similar pace. However, in the core class this was definitely not the case, as during the board problems I found myself monopolized by a few individuals who needed to be talked through the process because seeing it done on the board was not enough. I would definitely do this again and I think it would be an interesting concept to have a senior math major team up with a faculty member to teach a class for an entire semester."

The Yearlings enjoyed watching a Fristie teach, while others liked the change of pace and perspective. Some even voiced surprise to find cadets who actually understood the material. As COL Myers put it, "thus admitting the possibility that some of them could understand it also." Instructor MAJ Samuel Wright felt that the experience would be valuable when the Firsties become Lieutenants.

The instructors and cadets involved felt it was a positive experience. It helped integrate our majors more completely into the department. And it helped bridge the gap students sometimes perceive between themselves and their instructors.

Counterpoint

MAJ (Chaplain) Carlos C. Huerta
United States Military Academy

"Good fences make good neighbors."
MENDING WALL by Robert Frost

Who would not defend the innocuous idea of the need to breakdown barriers and build bridges? It only seems too obvious that in today's world we need to have more communication, need a broader exchange of ideas, and need to limit professional self-isolation as much as possible. In mathematics, this implies the need to explore as many other disciplines as possible to seek new applications to mathematical ideas. This is what many articles in this issue are saying but is that the whole story? Here we would like to explore whether in our bridge building if we have not perhaps built some bridges too far.

The question arises whether we as mathematicians can end up building bridges and tearing down walls that focus us out of existence. Bridge building is fine and can prove to be mathematically productive, but why does it often appear that it is the mathematics departments that "have to" build the bridges. If the truth were told, most of us did not become mathematicians to study physics, biology, or computers. We became mathematicians to do mathematics. Surely studying these subjects can be a part of doing mathematics, but they are only one small tool in the arsenal of a good mathematician. I would love to perform a study of how many mathematics departments have changed their curriculum to follow the flow of what the physics department was teaching versus how many physics departments have changed their curriculum to adapt to what the mathematics department was teaching. If we did such a study we may find that mathematics departments are far more "accommodating" than many other departments.

Perhaps our willingness to cross disciplines and build bridges can be interpreted as a loss of vision of who we are and where we need to be going. One can argue that there is something inherently wrong in trying to inspire and motivate students to learn and study mathematics by reaching largely outside of mathematics. We are all aware how important the flow of ideas from other fields is to the creative mathematical process, but perhaps we are not using these ideas to create mathematics as much as we using these other disciplines to justify our existence to our students and peers. If we need physics to inspire a future mathematician, do we have a future mathematician or future physicist? Do our students sense that mathematicians are apologists for their profession?

We as mathematicians know all too well the importance of a definition. A good definition can take you far, whereas a poor one can leave you crashed and burning on the field of logic. If we were to write, both individually and as a community, a definition of a mathematician, we would be surprised at the disparity of results. This disparity can be viewed as a good thing, as an indication of the diversity of who and what we think we are. Yet this diversity can also be viewed as an indication that we do not really know who we are and how we fit in this world of rapidly advancing technological based values. Are we only as good as our applications and use to technology and science, or is there something more important to what we do than that? Is there something that makes mathematicians view the world differently than others? If there is, what is that defining quality and how do we teach it to our students, assuming you can.

Take the concept of mathematical proof. Mathematics without proofs would fall into the category of "Alice in Wonderland". It is, unquestionably, at the core of what we do.

Proof, in its best instances, increases understanding by revealing the heart of the matter. Proof suggest new mathematics. The novice who studies proof gets closer to the creation of new mathematics. Proof is mathematical power, the electric voltage of the subject which vitalizes the static assertions of the theorems [1].

Yet, as important as this art is to mathematics, our recent attitudes in teaching have relegated the teaching and exposure of this art to the dustbin. We have come to accept the almost absence of proofs in our undergraduate mathematics courses, particularly those that service other departments. We have come to accept the, perhaps erroneous view, that engineers, physicists, and the like are not interested in how we do proofs; they only want to learn the manipulation of the symbols to solve their own professional problems. With today's generalized mathematics courses designed for the money paying audience, mostly non-mathematicians, the study and art of doing the "proof" has fallen by the wayside. We can see this in many calculus courses that have been sterilized and purged of all items that may resemble proofs. We do this at a great risk.

The inability to communicate proofs in an understandable manner has plagued students and teachers in all branches of mathematics. The result has been frustrated students, frustrated teachers, and, oftentimes, a "watered down" course to enable the students to follow at least some of the material, or a test that protects students from the consequence of this deficiency in their mathematical understanding [2].

Have we come to this sad state of affairs as a consequence of our bridge building? Is it as Morris Kline stated, "The greatest threat to the life of mathematics is posed by the mathematician themselves…."? [3].

We often speak of professional isolation as mathematicians and think we have discovered something new. It seems that we think this isolation is a bad thing, something to be avoided. Mathematical isolation is as old as mathematics, or even art itself. Gian-Carlo Rota said it well when he stated that, "A mathematician's work is mostly a tangle of guesswork, analogy, wistful thinking and frustration…." [1]. The ancient Greeks formed their mathematical societies not to build bridges but rather as vehicles to gather isolated practitioners together to practice their art together. I often think of Schroedinger's remark about his development of quantum mechanics when he says that it was one of the loneliest times in his life. In all creative endeavors, the creator feels the isolation of the creative process; mathematics is not any different. To try to remove this isolation is to perhaps affect the creative mathematical process in a negative fashion. Rather we must somehow come to grips with this sensed feeling of isolation and not try to remove it by becoming something we are not; we are not physicists, economists, or computer scientists.

Much has transpired to the mathematics curriculum over the last decade in response to the need for cross discipline collaboration. Some of it has been good and some not so desirable. We have looked in many places for “le fix” and it has been allusive to us as Fermat’s last theorem. As Fermat’s last theorem, the solution will not lie with a computer, or in another discipline, but in us. We have been reforming and changing how we do business for a long time in hopes of explaining to the world who we are and what we do.

It must, however, be admitted that the particular type of intellectual discipline obtainable from mathematical study on its formal, systematic, and logical side, is in considerable danger of becoming temporarily sacrificed during a too extreme swing of the pendulum of reform [4].

This was written in 1908, and though much has changed little has changed.

A series of questions present themselves in what we need to ask and think about as mathematicians and mathematics departments. Do we feel professionally isolated because we no longer know who we are? Have we bent our science and art to the whims of others to the point where we no longer have vision as to where we should be going as a community or teaching to the next generation of practioners? Do we often judge and explain the importance of mathematics to our students and contemporaries in other departments by its use by others and not by the joy of its creation and discovery, or beauty of thought? This are some of the questions that we must come to grips with, not just as private practioners of this ancient art, but also collectively, as mathematics departments.

The present mathematical fad is to seek ways to build bridges to other disciplines. This, in and of itself, is not a bad thing. But beware oh bridge builder. Bridges need firm foundations to be successful bridges. They must have a place to come from to have a place to go. Let us make sure that our side is secure.

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