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MATHEMATICA MILITARIS

CONTENTS

EDITOR'S NOTE

Welcome to the Spring 2010 Issue of Mathematica Militaris. Our theme for this issue is, "Innovation in the Classroom." As you all know, our classrooms tend to have some striking differences from the average college mathematics environment. One advantage of teaching in a service academy is the ability to highlight direct applications of the mathematics that we are teaching and to do so using examples that are frequently relevant to the future careers of the students in our classes. Moreover, class sizes are typically very small and this fact allows us to experiment with different approaches to group work and have more personal interaction with individual students than is common at a large university. In this issue, we have a collection of articles that illustrate these and other perks that we enjoy in our teaching environment.

First, Keith DeGregory (USMA) presents his unique approach to teaching applications of multivariate integration. This is followed by an article from Robert Block (USAFA) on the use of face to face assessment in a mathematical modeling course. Next, Christopher Smith (USMA) explains his use of formal groups to encourage student collaboration in the statistics classroom. Michael Courtney then updates us on the status of the evening tutoring program at USAFA. Finally, we end with a piece by Kyle Caudle (USNA) that illustrates how statistical simulation can be used to bring military applications into the mathematics classroom.

Over the past year, it has been a true pleasure to serve as the Editor-in-Chief of Mathematica Militaris. I have had the opportunity to learn about some exciting new developments at the service academies as well as gain a new appreciation for the heritage and traditions within our mathematics departments. I pass the torch of this position to Dr. Csilla Szabo, who I know will continue to improve on our efforts over the past two years to renew and reinvigorate the publication of this bulletin. Be on the lookout for the next Call for Papers from her!

Operation Order MA205: Integrating the Threat

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HOW do you teach calculus to students, from here on referred to as cadets, at the United States Military Academy? By giving them a military operations order (OPORD), complete with a mission statement and commander's intent, and then requiring them to analyze and execute the order. This was my out-of-the-box solution to covering double integrals during my last two semesters teaching Integral Calculus (MA205) as an active duty major and assistant professor at West Point. The objective of the lesson was for the cadets to apply the concepts taught in previous lessons on something tangible and of interest to them. Furthermore, they were to work as a team, practice their briefing skills, and as an added benefit realize that math can show up anywhere, even in military operations.

Obviously, I don't teach every lesson via means of an OPORD, nor is doing so a part of our mathematics curriculum at West Point. In fact, the idea came to me my first summer when I was assigned to teach a practice class on double integrals over general regions to my fellow incoming faculty. I had zero teaching experience at this point, so during one of our many workshops on pedagogy, I posed the following question to senior faculty, both military and civilian: "How do you get cadets excited and involved in learning something so clearly abstract as double integrals?" At the time I think I was looking for an answer such as: "Oh, that's easy you do *a*, *b*, and *c*." To my dismay I wasn't handed the answer on a silver platter; the answer I received was more like "It is different for everyone; you have to find what works for you." As I look back, I realize the answer I received was meant to be vague – it is necessary for new instructors to discover it for themselves; it is also a good way to bring fresh ideas to the table.

I spent the next few days developing and working on my Super Bowl of lessons for my first semester as a college professor. This lesson would be held two thirds into the semester during a Problem Solving Lab (PSL) immediately following lessons on double integrals over rectangular, polar, and general regions. The audience comprised of second year cadets with two semesters of undergraduate mathematics, two summers of military training and one and half credits of military science classes under their belts. I also had the fortunate opportunity to practice this lesson during that first summer on my colleagues. Leading up to this lesson the course had been kept abstract and most of the cadets still wrestled with how these concepts might ever be useful to them outside the classroom, much less in their military careers. They had no idea what was in store for them during that PSL as I maintained the advantage through the element of surprise. This is how the lesson went down:

When the cadets made their way into class on this particular day there was no teacher waiting eagerly to impart knowledge on them, rather they found a short message scrawled

on the chalk board with an envelope taped next to it and in the middle of the room on the floor was a terrain model for a military operation. The message stated that the platoon leader of 1st Platoon, A Company (1/A), had sustained battlefield injuries and had been evacuated to the rear. On the outside of the envelope was one of the cadet's names with his or her cadet rank replaced by the rank of Sergeant First Class (SFC) - a rank held by platoon sergeants in the regular Army.

Figure 1: SFC Rank

The newly appointed platoon sergeant (PSG) pulled the envelope from the board and with curiosity opened it to find a warning order (WARNO), a roster breaking the class into two squads and a platoon headquarters (HQ) squad, and a handful of various enlisted and NCO subdued pin-on rank designated for those in squad leader and special positions. Fortunately, this particular lesson fell on a Friday, a day on which the cadets and military faculty wear the Army Combat Uniform, lending itself nicely to the military scenario. After the PSG divided the class into the appropriate squads and the new squad leaders had affixed their Staff Sergeant rank onto their uniforms, the PSG proceeded to read the WARNO to the platoon as depicted in Figure 2.

The purpose of the WARNO is to provide subordinate units with enough information to begin preparations in absence of leadership or the full operational order; remember, there was no instructor in the room at that point. A few minutes into the regularly scheduled class hour I wandered into the room, playing the role of a brand new platoon leader assigned to his first unit in the middle of combat operations, and asked if this was the location of First Platoon. I broke the ice with some references to big-screen blockbusters: first I informed them that their old platoon leader, Lieutenant (LT) Dan, had lost both his legs, but remains in good spirits and sends his best. Next, I mentioned to a Private Ryan (which I actually had in one of the classes) that I bumped into a squad that was out searching for him. With everyone's complete attention, something that rarely occurs in one of my more standard lessons, I directed them to the terrain model and, with the intensity of a seasoned lieutenant, proceeded to issue the OPORD in grand military fashion.

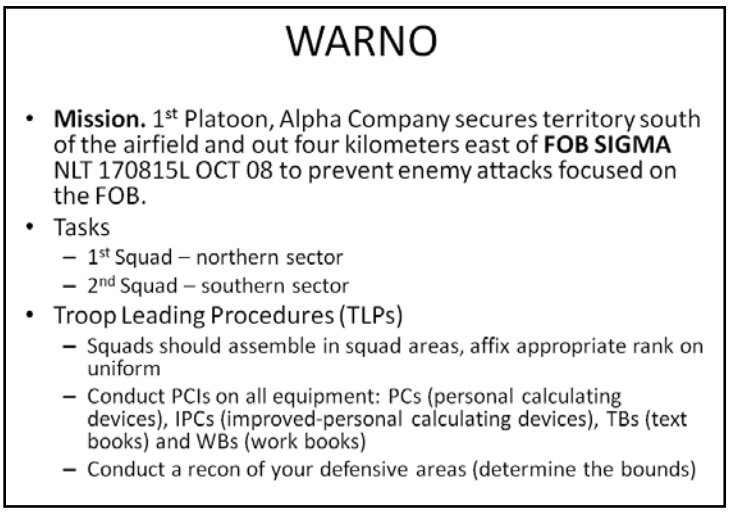

Figure 2: Military Style Warning Order for the Operation

Figure 3: MAJ DeGregory issuing OPORD as a LT

These cadets already had a fair amount of exposure to military training and operations; therefore, I was confident the five paragraph operations order would not be cause for confusion. What I was unsure of is whether or not they would be able to link this operation to what they were learning in class. My hope was that they would see how the concepts they had recently studied could be used to model and solve the given problem in the scenario so they could accomplish their mission.

Here was the mission and situation: 1st Platoon, Alpha Company, located on Forward Operating Base (FOB) Sigma, had to counter the enemy threat that was staging attacks from the east. The entire scenario could be modeled mathematically, some of these tasks included:

- determining mathematical equations for sector boundaries,
- determining a function to model the threat density,
- constructing double integrals over various regions,
- calculating the total number of attacks coming from each respective sector,
- calculating the center of masses of these attacks, and
- calculating the ratio of soldiers to attacks per square mile for each sector.

I provided a hard copy of the operational graphics (Figure 4) to the platoon leadership and in addition I taped out the graphics on the floor (Figure 5) for reference when I briefed the operation order. The northern and southern sectors comprised what a mathematician might characterize as a two general regions and a rectangular region minus two polar regions, respectively. The platoon headquarters was tasked with modeling the threat based off of descriptive intelligence provided by the S-2 (Intelligence Staff Officer) in addition to synchronizing the efforts of the two squads. Each of the squads were given specific instructions to calculate the expected number of attacks for their sector as well as the location for the center mass of those attacks. Furthermore, each squad was expected to provide a back brief of their analysis and results to the platoon leader 10 minutes prior to start point time, which for this mission was the end of the hour.

The platoon leader's intent and assigned tasks were explicit; what was not stated, and had been purposely omitted was the process to solve the problem. Only 45 minutes of class time remained at this point, thus I ended my major role in the scenario by suggesting they work speedily and efficiently. From there I stepped back and observed what I had put into motion. The very first time I executed this lesson I admit I was anxious and worried that the cadets would all stare at me dumbfounded with no clue how the foolish skit put on by this first year instructor had anything to do with math. Fortunately, what transpired the rest of the hour far surpassed my expectations for that PSL.

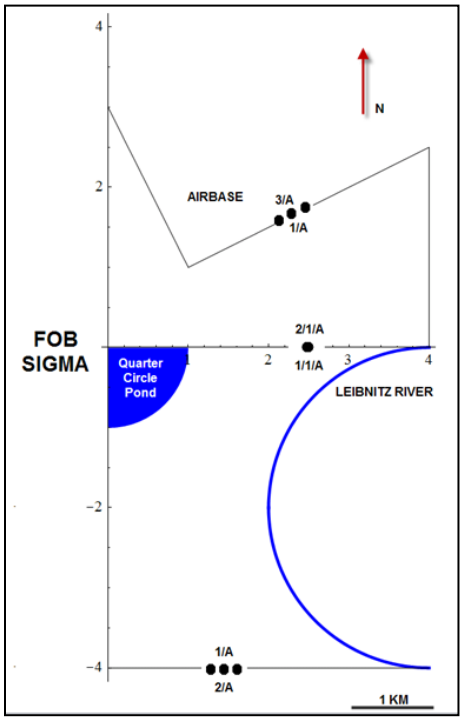

Figure 4: Operational Graphics

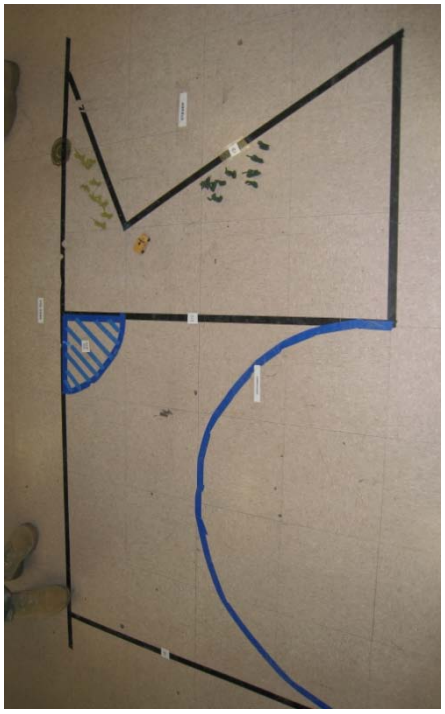

Figure 5: Terrain Model

The first thing I observed was the seriousness with which the cadets took their roles. The squad leaders quickly gave preliminary marching orders to their respective squads and then met in a huddle with the PSG to devise a game plan. Once the leadership decided on their plan of attack, the squad leaders returned to their respective squad areas to supervise the ensuing analysis. Meanwhile, the platoon HQ squad diligently tried to figure out the threat density function and quickly disseminated it once they realized it was critical information required by squads. Over the next half hour I observed some of the best collaboration and teamwork I had ever seen in the classroom in my few months of teaching.

Ultimately, the modeling consisted of the platoon headquarters section developing a threat density function (number of attacks per square mile) which is described as being zero at the edge of the FOB and increased linearly to the east where there were 4/3 attacks per square mile along eastern boundary of defensive sector (4 miles out). Thus the threat density function is

$$\rho(x, y) = \tfrac{x}{3}. \qquad (1)$$

Each of the squads needed to develop equations for their boundaries and then use those equations for the limits of integration for the various regions using the threat density function as the integrand. The resultant value would represent the total number of attacks (or mass) for that sector. There are a number of ways to set up the double integrals; here is one example for each respective sector:

$$\text{North: } m_N = \int_0^1 \int_0^{-2x+3} \rho(x,y)\, dydx + \int_1^4 \int_0^{0.5x+0.5} \rho(x,y)\, dydx, \tag{2}$$

$$\text{South: } m_S = \int_{-4}^0 \int_0^{4-\sqrt{4-(y=2)^2}} \rho(x,y)\, dxdy - \int_{-\pi/2}^0 \int_0^1 \rho(r\cos\theta, r\sin\theta)\, rdrd\theta. \tag{3}$$

After solving for the total attacks in each area, the squads then had to calculate the center of mass in each sector by first determining the moments about the x-axis (M_x) and the y-axis (M_y). The following are the moments for the northern sector:

$$M_x{}^N = \int_0^1 \int_0^{-2x+3} \rho(x,y)\, ydydx + \int_1^4 \int_0^{0.5x+0.5} \rho(x,y)\, ydydx, \tag{4}$$

$$M_y{}^N = \int_0^1 \int_0^{-2x+3} \rho(x,y)\, xdydx + \int_1^4 \int_0^{0.5x+0.5} \rho(x,y)\, xdydx. \tag{5}$$

The squads were then expected to use the moments to determine the precise center of mass $(\bar{x}, \bar{y})$ for the attacks. $\bar{x}$ and $\bar{y}$ for the northern sector are:

$$\bar{x}_N = \frac{M_y{}^N}{m_N}, \bar{y}_N = \frac{M_x{}^N}{m_N}. \tag{6}$$

Finally, they had to ensure that they maintained a 2:1 soldier to enemy attack (per square mile) in each squads sector. Their employment or recommendations to the PL would have to follow this intent.

After a successful lesson of modeling in teams and briefing their results I concluded the lesson by reassuring them they most likely would not have to actually take an integral as a platoon leader, to which sighs of relief followed. As for meeting the original objective of finding something tangible and of interest for which the cadets could apply the concepts covered in class, a survey taken a year later indicated that 86% of the 21 respondents recalled the lesson and of those, 94% stated they enjoyed the class and saw the relevance to the real world. The following is a representative of the comments from the cadets:

- *"The OPORD class finally put a real-life application that had a purpose to using integrals."*
- *"The OPORD class used double integrals to make a threat assessment of the area surrounding the base; allowing the commander to plan accordingly."*
- *"The OPORD class used double integrals to make a threat assessment of the area surrounding the base; allowing the commander to plan accordingly."*
- *"The OPORD class was a successful way of integrating our work in MA205 with a future job application."*
- "It was a chance to use math creatively in a military context, something we don't always have the opportunity to do."

As to long term retention of the mathematics, 56% correctly identified the lesson as an application of double integrals over regions, which is a significant considering half of my cadets went on to major in one of the humanities.

The lesson was a great success and fun for all six sections with which I executed it. Given the opportunity to teach integral calculus again I wouldn't hesitate to use it – and not because all lessons at a military academy need to be taught in a strict military format or manner. On the contrary, most lend themselves to more traditional classroom teaching techniques. However, I hope my story encourages all teachers, military and civilian, to take the extra time to create at least one lesson that is "outside the box" and make room for it in their schedule. I believe your students will enjoy the benefits as much as mine.

Face to Face Feedback to Spur Reflection and Develop Academic Success

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Introduction

I WILL discuss a feedback process used at the United States Air Force Academy in a course with a small class size where cadets are asked to reflect on their academic performance, assess their grade in the course, and to offer suggestions on how to improve their performance in the course. The purpose of this paper is to describe the feedback approach, how the cadets (and instructor) behavior changed, and to assess the approach. The thesis of this paper is that cadets will benefit from face to face assessment of their academic performance and their approach to learning. The feedback session includes a portion of time for the instructor to ask questions of the cadet and to discuss perceptions of how the cadet is performing and assess the effectiveness of the cadet's current learning approach. The cadet is provided the opportunity to assess not only their actions, but those of the instructor and peers. The cadet is asked to determine what grade they would award their own performance. A discussion of the instructor's assessment of the cadet is the starting point for reconciling any differences. It is my hope that this approach may be evaluated by other instructors teaching courses with small class sizes at the service academies and adopted as they see fit.

Small Course Feedback

Belton, Gould, and Scott [1] describe their course development process as an effort to develop a student that is a reflective learner because "an actively reflective learner is more likely than a passive learner to develop into a reflective practitioner." One way to encourage reflection is to provide face to face feedback to students. This discussion should be structured to include an opportunity for the student to reflect on academic performance. In my mathematical modeling course (23 students), I was able to design

my course so that I had an opportunity to provide feedback and stimulate reflection on the part of the students.

At the mid-point of the semester, we also completed a progress report on students that showed their grade in the course up to that point. My course was a junior and senior level major's course (Operations Research 411, Topics in Mathematical Programming). The goal of the course is to introduce both the theory and implementation of mathematical modeling and to develop analytic and communication abilities in the cadets. We investigate linear and non-linear programming and other optimization applications.

The course grading was based upon the following assignments listed chronologically in the following table:

Graded Events	**Points**
Homework Set	75
Quiz 1	75
Quiz 2	75
Quiz 3	75
Case Study 1	80
Quiz 4	75
Case Study 2	100
Case Study 3	120
Quiz 5	75
Final Exam	250
TOTAL	**1000**

The feedback session was not a graded event, and was conducted after the first case study was completed and graded. This occurred at the mid-point of the semester but before a majority of the points had been awarded in the course. The cadets had taken three quizzes but still had two remaining quizzes in addition to two remaining case studies and a final. At the mid-point of the semester, we also completed a progress report on students that showed their grade in the course up to that point.

The cadets were expected to review their academic work, assess what grade they would award themselves, and reflect on their performance to date. They were asked pointed questions as to how they prepared for the different types of assessments (homework set, quizzes, and case study). They were also asked to think about what went right and what went wrong in their performance.

The cadets were encouraged to make suggestions about the course that would potentially improve their performance and enjoyment of the course. They were also allowed to critique the instructor, the course content, and course design. Each interview lasted approximately 15 minutes. I also asked cadets to tell me what they planned to do to improve their performance and asked what I could do that would improve the course.

Results and Conclusions

I was amazed at how valuable this process was for not only the cadets but me as well. I expected the cadets to complain about the hard and difficult material; but, they did not do that. Fetterman [2] describes how senior staff members and faculty rate themselves unrealistically high until they are asked to document those ratings then more realistic ratings are selected. I expected the cadets to rate themselves higher than I did with regards to grades. But in 19 out of the 23 interviews the cadet assessment matched my suggested grade exactly. In 3 out of the 23 interviews, the cadets gave themselves lower grades, and in one of the sessions the cadet gave themselves a higher grade. I was also able to tell each cadet face to face my perceptions of what they were doing that was both good and bad.

Conversely, the cadets were able to tell me what they liked about how I ran the course and what they didn't like. Based on their feedback, I spent a little more class time lecturing on some of the harder topics and let them do more examples outside of class. I also expanded my use of real-world examples that applied to our subject material because they told me that helped the cadets make connections between the theory and the application of the theory which improved their learning.

The cadets told me at the end of the semester they appreciated that I had listened to their feedback and made changes that they benefitted from. The cadets' performance in the course was higher than previous semesters based upon the results of a similar final exam and anchored exam questions.

Anonymous end of course critiques are conducted at the end of every semester. In my course, 19 of the 23 cadets completed the critiques. The cadets rated the course on a six-point scale (higher being better). Activities in the course were rated as a 5.4, more than one standard deviation higher than other courses at the Air Force Academy. Course feedback was rated a 5.3, also more than one standard deviation higher than other courses at the Academy. Finally, the overall satisfaction with the course was 5.3, again more than one standard deviation higher than other courses in the Math Department and other courses in the Basic Sciences Division of the Air Force Academy.

Every year as I assign grades I have always felt there was more I wanted to tell the cadets than just what grade was awarded. I wanted to compliment strengths and provide suggestions as how to improve weaknesses. I also wanted to hear from the cadets and have a dialogue with them regarding their learning as suggested by McMahon [3]. Using a mid-term feedback approach, I was able to sit down with each cadet one-on-one and provide feedback that allowed them to reflect on and improve their academic performance before the semester ended. I recommend this approach for not only this course but any course with a small class size.

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A Collaborative Learning Experiment

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Introduction

IN the Army, an example of leadership is empowering subordinates to accomplish missions, while supervising their work. I believe this approach has some relevance in the classroom, especially for instruction at the United States Military Academy or similar military academies.

To that end, I wanted to determine any benefits for cadets from being a member of a group (complete with contracts of expectations). Specifically, I wanted to see if it affects the student's usual level of preparedness for class discussion, ability to cooperate together in teams, and individual class participation. The classroom procedure that I put in place used collaborative learning techniques to try to improve understanding and confidence in the concepts of my math class. I found that these collaborative or group learning techniques helped with participation in class, created somewhat of a chain of command and cadets looked on the techniques favorably.

Classroom Procedure

In the course, MA206: Probability and Statistics, the fall semester is our off-cycle term. The majority of the cadets we see during the fall semester are advanced math sophomore cadets. Instead of taking the usual four classes of core math, they have taken an accelerated three class core math track. MA206 is the last class in their core math sequence. Additionally, about a third of the students are junior cadets who either couldn't take MA206 during their spring of sophomore year or failed either MA206 or another math class and are behind in their sequence. There were 207 students in the course, and sections were ordered by GPA within each hour. There were high GPA sections and low GPA sections. I had two sections with an average GPA slightly above the mean and two sections with average GPA slightly

below the mean when compared with all cadets in the course that semester. The relevant statistics are listed in Table 1.

	B Hour	C Hour	G Hour	H Hour	Course
Number of Cadets	18	15	18	15	207
Average GPA	B+ (3.43)	B- (2.74)	A- (3.69)	B (3.08)	B (3.18)

Table 1: Statistics of my hours and the course as a whole. B and G hours have a GPA slightly above the course

During the academic year 2009-10, in the MA206 class I taught, I assigned formal groups in my classroom. By assigning groups that the cadets would work in for the semester, I ensured that cadets no longer had to document help received from another member of their formal group, according to the West Point Dean's Documentation of Written Work [1]. After conducting some research, I found that most effective groups are formed by the instructor, using some method, not by allowing the cadets to pick their own groups [2]. With this in mind, I formed the groups by looking at the GPAs of the cadets in my classes. I separated them into three tiers of equal numbers (or as close as possible to equal numbers): upper GPA, middle GPA and lower GPA. Each group of three cadets within the section was formed by choosing one cadet from each of these tiers.

The groups were assigned for the semester and were required to complete a contract of expectation that they all signed. The contract they signed included the rules by which the group members would interact with each other, a statement that on-time work is a standard of the group, etc. In addition, I assigned seating in the class, so that the groups would sit together. Within the groups, I had them assign themselves one of three jobs for the functioning of the group, and had them include these assignments in their contract. The jobs included a coordinator (or team leader), recorder (typist of final documents) and checker (double checks solutions, turns in assignment) [3]. They remained in these jobs for the duration of the semester.

I used the groups not only for project partners, but also for group work in class. I frequently gave board problems or problem sets that I allowed the cadets to work in their groups either at the chalkboards or at their desks, but working together. During these times, I would leave the classroom and encourage discussion within the groups. I would leave so that groups would have to encounter problems together and work together to solve them, not just raise their hand and ask at their first struggle. In addition, if any group member was absent from class and I passed out board problems, it was the responsibility of the group member(s) present to get a copy of the board sheet for their absent group member. This allowed them to have responsibility within their groups and created an effective support group when they were absent or didn't understand something from class.

I think it's important to properly incentivize the groups. The groups must feel there is an incentive or "carrot" for positive activity/function and a drawback or "stick" for negative function. The "carrot" for each group was that for individual evaluations (one course quiz, and two course exams in the course), if the group's average letter grade in the assessment

was higher than the group's average letter grade of their GPA at the start of the semester, they earned 10% of the evaluation's worth in bonus points. For example, the WRIT in the course was worth 50 points. If a group's average GPA grade before my class was a B and their group's average letter grade on the WRIT was B+, they earned 5 bonus points. The course only had 1000 points, so the prospect of earning a total 30 bonus points (course quiz – 5, first course exam – 10, second course exam – 15) was a bonus of 3% to their overall grade. This amount of bonus points is not really enough to create an unfair advantage between other classes with other instructors, but enough that the cadets generally feel that the points are worth working together in their groups.

On the other hand, there is a consequence for non-performers within the group. If the group decides that a member of the group is violating the terms of the contract they all agreed to, they have recourse. If a group is non-performing because of the actions of a member of the group or if the group is simply dysfunctional, they can request a meeting with the instructor. After discussing the situation with all members, if a solution cannot be agreed upon, I can, as the instructor, "fire" the non-performing group member. This means that the "fired" member must complete all remaining group assignments (we have two projects in the course) individually or be added onto another group (with instructor and new group members approval, and new contract). Therefore the "stick" is the potential of doing more work and doing it by themselves.

Once the partners were assigned, I noticed that the class structure emphasized some of the underlying culture within the academy. There was, in effect, a chain of command set up and one that I could use as an instructor. I didn't feel like there were 18 individual students in my class, now there were up to 6 teams of three. I only really had to get a gauge of 6 teams and see how the team was doing with concepts. Additionally, when cadets would e-mail or tell me that they were going to be absent for a class, I offered additional instruction (AI), but told them that the first thing they should do is coordinate with their team. The reason for this is that their teams (or select members within their teams) are probably more accessible to them than I am since they all live together in the barracks and are only a short walk away during many hours of the day. This created an additional resource that cadets could use, rather than running to me for AI each time they didn't understand a lesson. It also gave them opportunities to display peer leadership within my classroom setting.

Analysis

There were a number of ways that I measured the result of the classroom procedure, but for the purposes of this paper, I will explore only the first semester that I used this method (AY09-10). Throughout the semester, I collected information about and from my students and monitored their progress throughout the course. One measure that I will intentionally not discuss is their individual grades on events. I don't have enough data to see how collaborative learning affected individuals and since I was using this classroom method on all of my sections, I don't have a control section to compare them against. In addition, I believe that, though instructors can help or hurt some with grades, much of the grade comes from the work the student puts into learning. I believe that on average, most instructors should not be

evaluated as an instructor based on the grades of their students. This belief affects the measures I track to evaluate this system. I do use section grades later to show that my group learning techniques do not disadvantage my sections, but not to show positive impacts.

During this semester, I asked the classes to take a learning style test, which scored them along a scale of 4 types of learning styles (Active /Reflective learners, Sensing/Intuitive learners, Visual/Verbal learners, Sequential/Global learners) [4]. This let me get a baseline understanding of my 66 cadets and the various methods of learning that resonated with them. The results are presented in Table 2.

Active	Balanced	Reflective
40%	42%	18%
Sensing	Balanced	Intuitive
37%	45%	18%
Visual	Balanced	Verbal
61%	30%	9%
Sequential	Balanced	Global
39%	51%	10%

Table 2: Learning styles of my sections AY09-10. Based on these results my cadets were largely balanced overall, but specifically, they are largely Active Learners or balanced between Active and Reflective Learners.

From understanding the learning styles of my cadets, I know that my sections were largely Active/Sensing/Visual/Sequential learners. Although it's interesting to know the various learning types my students, specifically, it's important that they are either Active learners or balanced between Active and Reflective. According to Soloman and Felder, "Active learners tend to retain and understand information best by doing something active with it," such as working on problems together and explaining them to one another [5]. When I formed the cadets into groups, I frequently gave them problems to work either at the chalkboards in class or at their desks with their group members. This gave them time in class to actively engage with each other and the new material. This should have helped those Active learners and those balanced between Active and Reflective to digest the new material.

Evaluation

Once I had a picture of the cadets I was working with, it was important to see and measure effects in the classroom. I collected a fair amount of quantitative and non-quantitative data to attempt to measure the effectiveness of groups in my classrooms.

One of the qualitative measuring techniques I used was taking notes on the respective responsiveness of groups in class throughout the semester. It provided me more of a historical reference that I could review and come up with a general notion of how the groups were doing in my classes. Initially, as one would expect, the groups did not necessarily jump to working together, but as the semester wore on, I've recorded in my notes that the cadets

found that they enjoyed group working time. It was a way of breaking up the traditional lecture time and turning the class and learning over to cadets teaching other cadets. There were some teams that were not functional, but after some counseling, I found that we worked out any issues.

One of the things that was not a specific intent of my classroom experiment, but I found as a byproduct, was the effect of a chain of command or at least a support group within the class. As the instructor, I was not singly responsible for getting absent cadets missed information, keeping up with cadets who were falling behind, or for organizing the class. I found that within the group, they would often know about the accountability status of another: "Sir, John's not going to be in class today, he took off for a choir trip section today," etc. Working with the groups helped to take some of the pressure off of me for making sure cadets were responsible for their own learning. Since they earned bonus points if they performed well on individual events, the teammates tended to help each other out more.

The final quantitative measure I used to make sure that I didn't disadvantage my cadets by trying the group experiment was to explore their final grades and compare them to the average final grade for the course. You can see these statistics in Table 3. I don't think it's reasonable to state that putting them into groups helped their final score because there can be so many other things that affect this grade, as I've mentioned earlier in this paper. Since I'm the only one in the course that worked with groups, if my groups were consistently below the course average, I think that would be an indicator of disadvantaging my cadets by using the groups. In looking at the results, I think their final grades indicate that using the group learning methodology in class did not negatively affect their final grades.

	B Hour	**C Hour**	**G Hour**	**H Hour**	**Course**
Number of Cadets	18	15	18	15	207
Average GPA (Quality Pts)	B+ (3.43)	B- (2.74)	A- (3.69)	B (3.08)	B (3.18)
Average Final Grade (%)	A- (90.8)	B- (82.89)	A (93.61)	B (85.73)	B+ (88.14)

The last method of evaluation was in the course-end critique. I specifically asked the question, "Please write any suggestions for improvements of groups based on working in them all semester." Since almost all of the suggestions were ideas to change the group experience, not to cancel the groups, I consider that a very strong positive message. The vast majority of the comments for improvement revolved around group members being able to pick their own groups. Although there were a few who didn't like the contracts, or who didn't like the groups, e.g.,

> *"I thought the writing of the contract was a bit ridiculous and unnecessary, the group system did not play a vital role in my development in the class…,"*

the majority of responses were favorable, e.g.,

> *"Small groups were very helpful…, I believe the groups helped me out greatly…, I don't have any suggestions because I had a great group…, the groups work well."*

In addition, when we asked "If you could keep one aspect of the math program the same, what would it be and why?", though most of the comments related to the course, some of their comments related to their groups, e.g.,

> *"Having groups that we can work together with…, the groups because they contributed a lot to my learning…"*

I think that because of the free response nature of the question, the fact that some cadets included their favorable comments about groups makes a powerful statement that they liked and learned from their groups.

Conclusions

On the whole, I think my classroom procedure worked very well. The cadets seemed to like and respond to it and it certainly made some aspects of teaching easier for me as an instructor. I've used this procedure for other sections I've taught here at West Point, continually trying to modify it based on cadet feedback. I think that this procedure is not the panacea for student learning, but it fits with my teaching style and a majority of the cadets' learning styles. Based on my experience and analysis of the data I collected, I think that using group practices in my classroom helped cadets both understand the material and gain confidence in their ability to learn and understand mathematics.

ACKNOWLEDGEMENTS

The idea for many of the specifics of this work came from a discussion and later e-mails with LTC(R) Barbara Melendez on or about August 2008. She gave me examples of contracts used in her classes and ideas on point structure within my class.

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The Evening Tutoring Center at the United States Air Force Academy

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Abstract: The United States Air Force Academy (USAFA) has opened an Evening Tutoring Center to provide after-hours tutoring every evening before class days. The center focuses on first and second year courses in challenging quantitative disciplines: mathematics, physics, chemistry, and engineering mechanics. Staffed exclusively by faculty-level instructors, the center offers extra instruction to all first and second year cadets. Early demand has been remarkable, and early indications are that cadets who visit the center for extra instruction in mathematics perform better than peers with comparable backgrounds.

Introduction

MAINTAINING historical standards of academic rigor entails unique challenges as competition stiffens for the best students, mathematical prowess of high school graduates declines, intercollegiate athletes face the same core course requirements as other cadets, and technological distractions such as internet gaming, social networking sites, and mobile communication devices challenge both character and work ethic. For several years, Student Academic Services at USAFA has offered coursework in essential study skills and college level reading, as well as a Writing Center to provide tutoring in writing and communication skills. This year, Student Academic Services at USAFA added a Quantitative Reasoning/Evening Tutoring Center to provide additional instruction in the evening (from 1800-2200 hours) in first and second year courses in Mathematics, Physics, Chemistry, and Engineering Mechanics.

The Quantitative Reasoning/Evening Tutoring Center is staffed by four faculty instructors (one in each discipline) who teach one section of a core course in the day and provide extra instruction during the evening hours. These four instructors are highly qualified, with three PhD's and one M.S. and a combined total of 100 years of prior teaching experience. Tutoring instructors new to USAFA participated in the New Instructor Training program in their respective departments, and each is a full participant in departmental matters related to the course they are assigned to teach. The Chemistry tutor is responsible for tutoring both the first year Chemistry sequence as well as second year sequence in Organic Chemistry. The Physics tutor is responsible for tutoring Physics 1 (Mechanics mostly) and Physics 2 (Electricity and Magnetism). The Engineering Mechanics tutor tutors the single, 200 level course in Engineering

Mechanics. The Mathematics tutor is responsible for Pre-calculus, Calculus 1, Calculus 2, Calculus 3, and Differential Equations. This article describes the philosophy, process, and early outcomes of tutoring Mathematics at the Evening Tutoring Center, as well as ideas for meeting the challenge of rapidly increasing demand for additional instruction after hours.

Philosophy

The main goal of the service academies is developing officers of character. Quantitative prowess and analytical reasoning contribute to winning wars, but character counts for more. The Dean of Faculty is keen on imparting ample contributions from both Athens (wisdom and knowledge) and Sparta (warrior ethos) to cadets, and due consideration is given to train cadets with the character needed to maintain principles of honor and their commissioning oath to support and defend the Constitution. This is no dichotomy: cadets who aspire to be officers of character must be willing to acquire the wisdom and knowledge necessary for the performance of their duties.

So while the educational philosophy of the tutoring center is customer focused (how can we help the cadet succeed in the topic at hand), the longer term focus is in preparing cadets who are responsible, disciplined, mature, and hard working in the service of our country. Key questions we ask ourselves are "Have I encouraged the cadet to grow in the habits and disciplines of a good officer?" and "Have I encouraged the cadet to take ownership of his assigned work as important preparation for future duties?"

Cadets utilize the Evening Tutoring Center for various reasons: some have scheduling conflicts with daytime extra instruction, some have weaker backgrounds in Mathematics, some have not yet learned the proper habits of mind and disciplines of time management needed for success. Most are still teenagers experiencing new levels of pressure and responsibility. They need encouragement and practical advice to rise to that challenge, along with the reminder that they are capable of tremendous accomplishment with the proper application of time and effort.

The role of the tutor is distinct from the role of classroom instructors. Classroom instructors have a dual burden: 1) they are teachers who impart knowledge and guide cadets as they accomplish the required learning objectives, and 2) they are gatekeepers who must ensure quality of learning by developing assessments that accurately reflect the level of student accomplishment. Classroom instructors bear both the carrot (promises and joys of learning the subject) and the stick (firm requirements and consequences if the cadet fails to meet course requirements). In contrast, tutors bear mostly the promises and joys. The tutor gets to be the "good cop" who sympathizes with realities of cadet life and academic challenges and comes along side to help the cadet meet the requirements. Tutors have the advantage of connecting the last few dots so cadets can see the picture clearly. Tutors get to be the voice that says "you can" when other voices, internal and external, are saying "you can't" or "you won't."

As with everything in life, success is conditional, but the tutor breaks down complex challenges into manageable portions so that cadets can see their way through the maze. Quantitative problem solving is analyzing the problem and planning a strategy more mature than formula roulette (hoping to get lucky by applying formulas without really understanding why a given formula is appropriate in a specific context). Before beginning to take a derivative or integral, one must consider how the derivative or integral is related to the broader problem, and one must consider why one has chosen a given approach from the array of possibilities.

Process

Earlier programs offered by Student Academic Services grew gradually as the word circulated that additional assistance was available in writing, study skills, and college-level reading. Open four hours each evening, Sunday-Thursday, The Evening Tutoring Center's initial plan was to focus on individualized attention in scheduled one-on-one tutoring sessions. In the first week of operation, it became clear that meeting cadet demand with available staff would require a combination of individual and group sessions.

In Mathematics, ten hours of course-specific walk-in sessions are scheduled each week, according to demand projected from consideration of assignments, graded reviews, and historical demand. These walk-in sessions are invariably group sessions with anywhere from 3 (ordinary homework help) to 23 (graded review the next day) cadets in attendance. The remaining 10 evening hours each week are used for individualized instruction for which students sign up using an on-line calendar (SharePoint) on a first-come, first-served basis. When the Evening Tutoring Center opens each Sunday night, nearly all of the individual time slots are already taken for that week.

Individualized tutoring is a favorite activity of tutoring center faculty. One gets to review cadet work and identify weak areas. One gets to know cadets well: where they are from, what they hope to major in, why they came to USAFA, what study habits they have built, and where they need additional encouragement. One gets to ask if they are satisfied with the tutoring services. Most of all, one gets the joy of seeing the light go on, observe cadets grow in confidence and ability over the course of the semester, and have cadets arrive beaming and announcing their recent triumph over the latest graded review.

Tutoring group sessions is a greater challenge. Up to groups of three or four, I can still develop a good sense for where each cadet is getting stuck and allow the cadets to work problems while I offer corrections, advice, and encouragement. Depending on the group, once there are more than 5-6, I shift to modeling good problem solving techniques at the board while keeping students engaged by asking them what the next step is and why. Occasionally, group work among students can be productive under the tutor's guidance, but one must be careful about the "blind leading the blind" and bad problem solving habits being propagated. Cadets love short cuts, and the tutor's job is to model problem solving methodologies that are well-considered, well-communicated, and easy to review

for errors if the bottom line doesn't make sense. Having cadets help each other is more productive in second year courses where there is more maturity to seek genuine understanding and less of a tendency to seek the shortest path to the "right answer."

Demand

Demand is dominated by Calculus 1 followed closely by Calculus 3. Demand for other courses is far below demand in Calculus 1 and 3. Reasons for Calculus 1 demand are obvious. Every institution has students struggle in calculus even though they have rigorous admission standards. Calculus 1 is the first college level math course for many cadets, and it requires a level of discipline and good habits of mind that many cadets are still developing. We attribute Calculus 3 demand to the fact that the Calculus 1 and Calculus 2 are designed as three credit hour courses geared for all majors. In contrast, Calculus 3 is comparable to multivariable calculus courses in engineering and science majors at top 20 schools and represents a significant step up in the mathematical maturity, quantitative reasoning, and problem solving skills compared with the first year Calculus sequence. It is harder for cadets to move into our multivariable Calculus course with a background of 6 credit hours in Calculus than it is for students at other institutions with 8-10 hours of pre-requisite Calculus courses.

In the first semester of operation (Fall 2009), the tutoring center served over 2500 cadets, over 800 in Math. On a typical evening, the tutoring center sees 2-4 cadets for individualized sessions in math, and 5-10 cadets for group walk-in sessions. Most students will make good use of the whole hour, if they come to an hour-long walk-in session or schedule an hour long appointment. Students are encouraged to come prepared and most show up with a specific list of homework problems they need help with and/or specific topics to discuss. Students needing less help will typically schedule only a half-hour individual session.

Outcomes and Discussion

Cadets served by the Evening Tutoring Center tend to have weaker backgrounds than the cadet wing as a whole. For example, the cadets enrolled in Calculus 1 who visited the tutoring center in the first two weeks to prepare for the first Fundamental Skills Exam (FSE),[1] which covers high-school Algebra, averaged 52.1%±2.9% on the Algebra portion of the departmental placement exam, whereas the cohort of cadets currently enrolled in Calculus 1 averaged 60.1%±0.6% on the Algebra portion of the placement exam. (A score of 38% is considered passing, sufficient for placement into Calculus 1. Higher scores are required for placement into Calculus 2 and 3.) However, outcomes of cadets who utilized evening tutoring services tend to be comparable with the cadet wing as a whole. For example, the cadets who visited in preparation for the first FSE scored an

[1]A more complete discussion of Fundamental Skills Exams at USAFA is found in: *Teaching Fundamental Skills at the United States Air Force Academy*, James S Rolf, Michael A. Brilleslyper, and Andrew X. Richardson, Mathematica Militaris, Volume 15, Issue 1, Spring 2005.`

average of 76.5%±4.6%, whereas the complete cohort of cadets in Calculus 1 averaged 78.4%±0.6% Consequently, cadets who used our services improved their score by an average of 24.4%±3.6% between the two events, compared with the complete Calculus 1 cohort who improved their average score by 18.3%±0.6%.

This is no surprise. Every math teacher can attest that cadets who receive extra instruction tend to do better than cadets who do not. Individualized attention can identify and address a cadet's specific struggles. The service academies are well-known for instructor availability. However, there is clearly additional need for extra instruction beyond traditional office hours. Intercollegiate athletes have mornings filled with classes and afternoons filled with athletic practice. Evenings are the only time they regularly have available for extra instruction. Earning 147 credit hours in 8 semesters, along with military training and duties, in addition to the required intramural participation also makes for full days for many other cadets. The academic call to quarters each evening provides a block of time when many cadets can make use of available extra instruction. Cadets, instructors, and course directors have all expressed appreciation for the availability of evening tutoring services in challenging quantitative disciplines. (MIT has had evening tutoring available for almost 30 years through their Office of Minority Education. Principally staffed by graduate students, their tutoring services are open to all students. I served as a Calculus tutor there while in graduate school, and their system serves a wide variety of minority and non-minority students.)

As word circulates regarding the availability of the Evening Tutoring Center, we expect demand to continue to increase. Additional increases are expected as cadets already using our services succeed and move on to courses currently under-represented in our demand profile (Calculus 2 and Differential Equations). Maintaining adequate service will prove challenging with the current staffing level. As budgets and staffing permit, we foresee several enhancements to meet this challenge.

The Writing Center already makes use of faculty volunteers to approximately double its tutoring manpower compared with their dedicated tutors alone. The Chemistry department was the first quantitative discipline to send faculty to augment the efforts of the dedicated Chemistry tutor. With the size of the USAFA Department of Mathematical Sciences, evening tutoring availability in Mathematics would be doubled if each department member volunteered for evening tutoring once every two months.

Another idea for better serving students with existing manpower would be to provide on-line availability to high-quality exemplar solutions for assigned homework problems or close analogues. Of course, the answers to many problems are in the back of the book, and answers are often posted on-line to selected homework problems. However, published answers and solutions are of variable quality, often overemphasizing equations rather than modeling sound problem solving. Potential benefits of this approach are limited, particularly for first year students, because at this stage of their mathematical development, many students are not yet sufficiently disciplined to focus and gain full benefit from even the highest quality written solutions. Rather than do their best on a

given problem until legitimately stuck, view the model solution to focus on the solution process, and then return to complete their own solution, students rely too heavily on written solutions.

Another option is to provide a list of topic-specific on-line resources. For example, the tutoring center is incapable of providing adequate resources to meet demand when a significant number of cadets miss the same lecture while traveling to an intercollegiate athletic event, and tutoring is not really a substitute for a quality lecture. As part of their Open Courseware materials, MIT publishes a complete set of on-line high-quality video lectures covering nearly every topic in differential and integral calculus, multivariable calculus, and differential equations. (See http://ocw.mit.edu.) While the MIT lecture style differs from the Thayer method favored at USMA and the similar hands-on style favored at USAFA, these lectures can be a valuable supplement for cadets who miss a specific lecture or whose learning style is better served by a traditional lecture format. There are also a set of high-quality Calculus videos published by the University of Houston (http://www.online.math.uh.edu/HoustonACT/videocalculus/index.html) that are more tutorial in style and may be better suited for students who are building on a classroom lesson.

In addition, a number of the mathematical articles in Wikipedia are of surprisingly high quality (given that anyone can edit them and many editors are anonymous) and can provide an explanation that is complementary to the textbook and classroom presentation. I have vetted a number of these entries and compiled a list of quality entries for students to use at my classroom sub-page of the Calculus 1 SharePoint web site. While existing internet resources can be valuable, I recommend careful vetting for quality as well as compiling an organized list of topics that correlate well with specific material in the course.

Cadet feedback indicates they benefit more from on-line video lectures than from on-line written material. In addition, tutors are often asked to model problem solving strategies for the same homework exercises repeatedly by different cadets. This suggests we can serve more cadets with limited manpower by producing videos that model well-communicated solutions to assigned exercises or close analogues. Cadets would probably prefer that we provide model solutions to assigned exercises, but there is probably more benefit in modeling close analogues and allowing cadets to do the mental labor bridging the gap between the close analogue and the assigned problem. Listing each on-line video tutorial according to textbook analogues would quickly guide cadets to the available assistance when they are stuck on a specific problem or concept.

In summary, early feedback indicates that the availability of evening tutoring will enhance student success in challenging quantitative coursework, particularly by better serving cadets who have scheduling difficulties meeting with their primary course instructors during daytime office hours. Meeting cadet demand for evening tutoring will be an ongoing challenge, but we have identified several avenues that should prove productive as we move forward.

Simulating Expected Value Calculations in a Military Setting

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Introduction

IN various war gaming scenarios, some consequences are so severe that military planners must completely rethink the battle scenario and start the planning process over. Because of the number of possible alternatives, these problems can have as many outcomes as a complicated combinatorics problem. Since this situation lends itself admirably to simulation, one can use this to show students how problems with very complicated closed form solutions can easily be simulated using a high level language such as Matlab or R.

One can project the expected outcome of a process by running a simulation repeatedly and averaging the results [2]. Monte Carlo simulation is the term that is given to this process of repeated simulation. Stanislaw Ulam, a Polish Scientist who worked with John von Neumann on the Manhattan project in 1946, was the first to use Monte Carlo methods when trying to determining the probabilities of winning a card game of solitaire. The phrase "Monte Carlo methods" was actually credited to Nicholas Metropolis who worked with Ulam and von Neumann [3].

Expected Value

The concept of expected value is one that is taught in elementary probability courses. The expected value is the value of a random variable that one would expect to obtain (on average) if a probability experiment were repeated a "large" number of times. In order to calculate the expected value, one needs to know the probability density function $f(x)$. Once the probability density function is known, the expected value is calculated by multiplying each possible value of the random variable by its associated probability and then summing over all values of the random variable [1].

$$E[X] = \sum_{all\ x} x \cdot f(x), \qquad \text{if } X \text{ is discrete}$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)dx, \qquad \text{if } X \text{ is continuous}$$

For complex scenarios, the probability density function must usually be constructed unless the scenario lends itself to one of the well known probability functions (e.g. Poisson, binomial, normal, etc.). Because of the speed of computers, exceedingly

complex scenarios lend themselves to simulation which can provide remarkably accurate results [4].

Basic War Gaming Scenario

As a very basic example of this process, consider a scenario where a missile must obtain 2 consecutive hits in order to destroy a target. Furthermore, assume that if a miss occurs, the system operator must stop the exercise and recalibrate the equipment. Once recalibration is complete, the exercise commences again but 2 consecutive hits are still required regardless of the previous hits. To simulate this, I will use an ordinary deck of 52 playing cards. An ace represents a miss and anything else a hit. Thus, the probability of a hit is 48/52 or 12/13. The probability of a miss is 4/52 or 1/13. The goal is to determine the expected number of shots required before obtaining 2 hits.

This scenario does have a closed form solution that is not too difficult to calculate. Let the random variable X be the number of shots required to obtain 2 consecutive hits. Clearly, the minimum value for X is 2. If we assume independence[1], the probability that $X = 2$ would be $(12/13)(12/13) = 144/169 \approx 0.852$. If $X = 3$, this would require a miss and then 2 consecutive hits. The probability of this would be $(1/13)(12/13)(12/13) \approx 0.0655$. For $X = 4$, there are a several possible scenarios: (Miss-Miss-Hit-Hit) or (Hit-Miss-Hit-Hit). Thus, the probability that $X = 4$ would be the sum of these 2 disjoint probabilities. $(1/13)(1/13)(12/13)(12/13) + (12/13)(1/13)(12/13)(12/13) \approx 0.0655$. For $X = 5$ there are three possibilities, (Miss-Miss-Miss-Hit-Hit), (Miss-Hit-Miss-Hit-Hit), and (Hit-Miss-Miss-Hit-Hit). The sum of those 3 disjoint probabilities is ≈ 0.0097. (Note: Although not specifically stated above, after obtaining 2 hits the cards are collected and reshuffled prior to starting the next simulation run.)

Continuing in this manner, the probability density function would be[2]:

X	$\Pr(X=x)$
2	0.8520
3	0.0655
4	0.0655
5	0.0097
6	0.0054
7	0.0011
8	0.0005
9	< 0.0001

[1] If independence is not assumed, this would require a conditional probability calculation which adds to the complexity of the closed form solution.

[2] It is acknowledged that the probability of anything beyond $X=9$ is not identically equal to zero, but is less than 0.0001.

From the probability density function we can calculate the expected number of trials necessary to achieve success:

$$E[X] = 2(0.8520)+3(0.0655)+4(0.0655)+5(0.0097)+6(0.0054)+7(0.0011)+8(0.0005)$$

$$= 2.2551.$$

The complexity of the probability calculations was due in part to the fact that it was possible to start over. In real world situations, we may also have situations that require a delay, but not a complete restart. Also, we have only considered 2 outcomes (Hit/Miss); most situations have many more possible outcomes.

Simulation Algorithm

For simulation, one needs only to run the scenario and count the number of times the scenario runs until success is achieved. The basic algorithm for the prior scenario would be:

Algorithm: Simple Missile Program

```
1:    m = 5000;                        # number of times to run the simulation
2:    for i =1…m do
3:        hitcounter ⟵ 0               #  initialize hitcounter to zero
4:        trialcounter  ⟵ 0            #  initial trials to zero
5:        while (hitcounter < 2) do
6:            shot = rand(52)          #  random integer between 1 and 52
7:            if (shot < 5)            #  shot was a miss
8:                hitcounter  ⟵ 0      #  reset counter
9:            else hitcounter ⟵ hitcounter + 1
10:           end if
11:           trialcounter  ⟵  trailcounter + 1
12:       end while
13:       trials(i) ⟵ trialcounter
14:   end for
15:   mean(trials)                     # print out the average number of trials to success
```

If we plot the individual mean value at the end of each simulation run, we can get a picture of the true expected value for the scenario. Figure 1 is a plot of expected value for the Simple Missile Program vs. the simulation run.

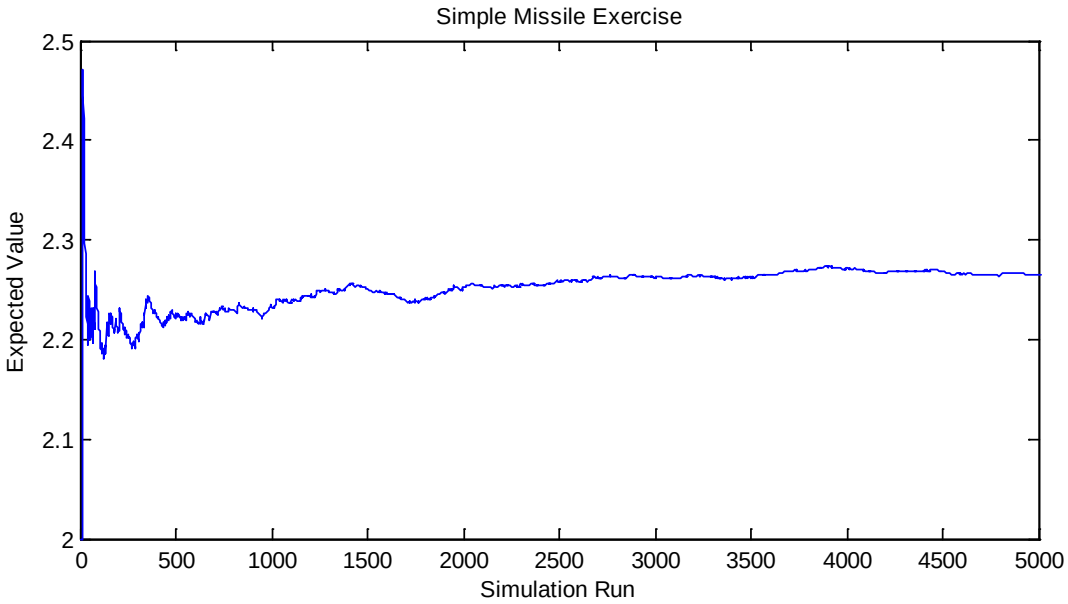

Figure 1: Expected Value for the Simple Missile Program

The average number of trials after 5000 simulation runs was 2.2342. The simulation ran in just under 1 second. Doing 10,000 or 100,000 simulations would be quite feasible and would produce even greater accuracy.

Conclusion

In the context of a basic probability course, accurately determining expected values for complex situations can become quite tedious. When trying to get students to understand various concepts, it is not necessarily conducive to learning to get bogged down in the details. Simulation can be a powerful tool to solve complicated problems, and can also give students a visual representation of some of the underlying asymptotics.

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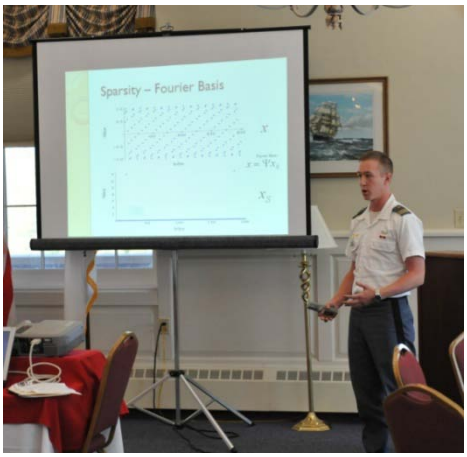
Service Academy Student Mathematics Conferences

Dr. Amanda Beecher and Dr. Edward Swim
Editors

THE 19th Annual Service Academy Student Mathematics Conference was hosted by the Coast Guard Academy in New London, CT on 16-18 April 2009. Four midshipmen from the Naval Academy, twenty three cadets from the Coast Guard, five cadets from the Air Force Academy and thirteen cadets from West Point spoke on their research on a

Welcome by CDR Michael Zamperini



CDT Andrew Carfang, USMA

variety of mathematics and operations research topics. In addition to the outstanding presentations by the students, keynote speaker Rear Admiral Thomas F. Atkin discussed his experiences with joint military branch operations, particularly in his new role as commander of the Deployable Operations Group. He highlighted how conferences such as SASMC create a forum for dialogue and friendship between the academies that will hopefully continue across branches of service. On the final morning, all participants

CDTs Hyne, Moyseowicz, and Ulmer, USCGA

were given a view into the Coast Guard facilities through a series of tours of the new Coast Guard Research and Development Center in New London, the Coast Guard Station

in New London and the Coast Guard Barque EAGLE. The efforts of the organizers and cadets of Coast Guard made this another successful and enjoyable conference.

Participants at the 19th Annual SASMC

The 20th Annual Service Academy Student Mathematics Conference was hosted by the Air Force Academy in Colorado Springs, CO on 22-24 April 2010. Four midshipmen from the Naval Academy, twelve cadets from the Air Force Academy and eleven cadets from West Point presented their research. The keynote speech was delivered by Dr. Brian Winkel, who is currently on leave from USMA and a Distinguished Visiting Professor at USAFA. His address focused on several entertaining examples of historical ciphers from American literature, including a famous example from the writings of Edgar Allen Poe and a not-so-famous novel of Jules Verne. The conference was capped off by a panel session on the role of mathematics in the careers of four military faculty members at USAFA. Once again, the organizers and cadets at our host academy provided superb hospitality to all that attended.

Participants at the 20th Annual SASMC

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Mathematica Militaris is a forum where faculty and students at each of the four service academies can publish their work, share their ideas, solve challenging problems, and debate their opinions. Anything mathematical (proofs, problems, models, curriculum, history, biography, computing) is in the purview of the bulletin. While the missions of the mathematics departments at the service academies are quite similar, each has a different means of accomplishing its goals. By sharing information, we will be able to improve our programs by learning from each other. Hopefully, through Mathematica Militaris, these programs can develop a common identity, gain recognition, and build an effective communication link.

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