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CURRICULUM CHANGE: WHY & HOW?

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EDITOR'S NOTES

Happy New Year! I became the *MM* New Guy upon rejoining the USMA faculty last summer. Kudos to my predecessor, LTC Mike Johnson, whose dedicated editorial work has set a high standard. My own journey in his footprints has led to this current issue. I hope you like it.

The lead article gives a State of the Union-style synopsis of USMA's current approach to its first-semester core mathematics course. Jointly authored by Lieutenant Colonel Heidenberg and his course leadership team, the article chronicles the impact of technology as a primary impetus for curriculum change.

Next is a story of adjustments to required & elective course offerings written by USAFA's Dr. Dale Peterson. Although his article pertains to a master's program, his central theme of "structure" versus "flexibility" is wholly valid in the service academy setting.

Then USMA author Professor Winkel reminds us of the critical human dimension as he recounts his thought-provoking experiences with curriculum change at Rose-Hulman Institute of Technology.

After that, we have the inaugural contribution from new Managing Editor Dr. Lee Zhao. He proposes an alternative to the standard calculus sequence for liberal arts students and also shares his successes using Hardy's *Apology*.

Next is a piece by Professor Lainis who draws upon his experience in USMA's Department of Physics to provide a multi-disciplinary point of view.

Finally, we have the nostalgic reminisces of Father Costa – USMA's resident differential-equations analyst.

Mathematica Militaris will maintain its biannual publication rhythm for the foreseeable future. As you read through this current issue, I hope you are inspired to share your own ideas, techniques, and strategies with your cohorts. Stay tuned for the next Call for Papers.

Be sure to visit our website for past issues: *http://www.dean.usma.edu/math/pubs/mathmil/* .

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Editor, *Mathematica Militaris*
Department of Mathematical Sciences
United States Military Academy
ATTN: MADN-MATH
West Point, New York 10996

Curriculum Change in a Changing World

Lieutenant Colonel Alex Heidenberg, Major Elizabeth Schott, and Major John Wasko, USMA, Department of Mathematical Sciences

The general educational goal of the United States Military Academy is to "enable its graduates to anticipate and to respond effectively to the uncertainties of a changing technological, social, political, and economic world." As the world continually changes, the corresponding academic curriculum must undergo continued assessment to effectively prepare students to achieve this goal. Technology's increasing employment in the use of problem solving has necessitated a dramatic change in the education curriculum, the role of the educator, and our methods of assessment. Graphing calculators and computer algebra systems provide the means for the students to quickly and easily visualize computationally intensive mathematics that once required significant time and effort. The Calculus Reform movement sought to improve instruction, in part, by taking advantage of these technological resources. Mathematical solutions can now be represented analytically, numerically, and graphically. In light of the technological changes, a necessary pedagogical shift occurred: from teaching mathematics to teaching mathematical modeling, problem solving, and critical thinking. Portable notebook computers, with even greater technological resources, have led to another reexamination of our goals and objectives. This paper looks at the initial response made at the United States Military Academy as it incorporated laptop computer technology into the classroom. Specifically, it will address the changes made in ***what we teach***, ***how we teach,*** and ***how we assess*** learning.

What We Teach

The assessment cycle begins with the articulation of goals and a set of objectives that, in combination, achieve these goals as an end state. The curriculum is established with these goals and objectives as its foundation. Well-designed assessments provide data and information that give insight regarding the level to which the student has mastered these objectives. The results of each assessment provide the impetus to repeat the assessment cycle.

The infusion of portable notebook computers into the classroom provided a justification to once again examine the goals and objectives of our curriculum. A review of the program goals for the math core curriculum led us to update these goals to incorporate the capability of the laptop computer. Solving complex problems with technology is now expected. At a deeper level lies the objective: namely, that simply solving an equation is no longer sufficient; students must integrate these skills into the problem solving process. Critical components of effective problem solving and analysis are exploration, experimentation, and discovery as part of a complete, comprehensive product. Laptop computers in the classroom facilitate such exploration as well as provide a useful medium to build and store a chronological library of their discoveries. "Real-world" problems, which is what we expect our graduates to solve, are complex and require technology to obtain a solution. Exploring mathematical concepts with technology to gain a better understanding and subsequently using technology in the problem solving process are skills we are trying to cultivate.

Problem solving labs were specifically designed to allow our students the opportunity to use technology to query, hypothesize, and explore solutions to more in-depth and mathematically challenging problems. Getting students to ask "what if" questions is an important part of the problem solving process that addresses the 'uncertainties' in a changing world and is a major goal of our curriculum. Guiding our students in the conduct of this sensitivity analysis is not without drawbacks. Using technology takes time that is ordinarily used in the classroom to undertake more concepts. The traditional trade-off between breadth and depth is exacerbated when technology is brought into the classroom on a daily basis. We continue to eliminate mathematical techniques from our core curriculum to concentrate on modeling and problem solving. It is our hypothesis that what we teach is not as important as how we teach. Real world problems do not exist as equations on chalkboards. Our instruction focuses on teaching students to be inquisitive, creative, and confident as they confront ill-defined problems.

How We Teach

Technological advances have forced changes not only in what we teach, but also the way material is presented and how and what students learn. For instructors, it has raised the question of how to inspire a conceptual and theoretical understanding of math concepts when these complex problems can often now be solved with a simple command. Previously, we required students to demonstrate some level of theoretical understanding prior to their implementation of technology to solve a problem. In incorporating laptops, we focused on using technology as a means of developing conceptual understanding.

With the shift in focus to mathematical modeling and problem solving, we concentrated on problem-based instruction. All topics were approached from a mathematical modeling perspective. Each new block or lesson began with the introduction of a new, often ill-defined problem. This allowed us the ability to continually practice problem solving techniques. Students were encouraged to conjecture solutions, estimate answers, develop solution techniques (which often required the exploration of new mathematical concepts), and then accomplish the key task of relating the solution back to the original problem.

Laptop computers have significantly increased our in-class technological toolkit: packages such as Microsoft Office give us editing and spreadsheet capabilities; computer algebra software, such as *Mathematica*, give us enhanced calculating and graphing tools; and the computer's hard drive allows us to store large quantities of information. We incorporated these tools first to explore new concepts and then to develop solution techniques or algorithms for the problem we were exploring. Once the solution techniques were developed, solutions to whole groups of problems could be quickly found.

By choosing open-ended or ill-defined problems, students are forced to carefully consider simplifying assumptions and how they change the problem. The speed of technology in finding solutions allows us to focus on exactly how the solutions we develop relate to the problems and gives us the ability to both discuss and explore a model and its solution's strengths and weaknesses. It allows for greater depth in sensitivity analysis and for the exploration of how a solution would change if the assumptions and/or facts of the problems changed. A simple numerical answer to a problem is no longer sufficient – solutions now require a discussion of the limitations of the answer and in what circumstances it is valid or invalid.

We found teaching with this approach included its share of challenges. Students (and often instructors) are uncomfortable with this approach. Classroom management proved to have many new challenging aspects. It takes considerable time to allow the students to struggle with ill-defined problems. It also takes considerable time to "teach technology." We found the best approach is to devote valuable classroom time to some basic technology "how-to." Without some initial instruction, the students stalled and did not progress very far out of class; too-detailed instruction in the classroom relieved the students from experiencing learning technology on their own. Additionally, learning is an "active sport." Students cannot learn to effectively use technology by watching demonstrations in the classroom. Assessments now required the effective use of technology in order to be successful in the class.

To utilize computer programs in class, the instructor must be prepared to dedicate time from the lesson on questions about the operation of the actual program itself. In a short class (55 minutes), this can be a significant amount of time and can slow down the pace of the class. To be effective on that day's lesson, we felt time must be properly allotted into the lesson plan. If not, you might get through the technology commands, but have insufficient time remaining to focus on the results of the technology and its applications – which is our ultimate goal.

The other approach to this is to try and push the cadets to teach themselves and make it part of the prep for class, which we have found is very successful in some cases and not so in others. Success with this method, unfortunately, depends more on the students' interest than anything else. Cadets who were interested and who enjoyed working on the computers quickly picked up the essentials and began exploring other commands. In other cases, cadets who initially experienced trouble got very frustrated and gave up, making them quickly fall behind. Without some basic classroom instruction, some cadets are forced to devote a large amount of time outside of class to learning these processes. This, in turn, impacts other material you may also be asking them to complete; thus, balance is always important.

How We Assess

All good assessments of learning involve a well-thought-out, well-conceived plan that involves multiple modes of assessment. Quizzes, graded homework, technical reports, and exams have traditionally provided sample data regarding what our students are learning. Paramount in the assessment process is determining what concepts and/or skills we want our students to learn in our core problem. We understand that "what we test is what we get;" therefore, we have adapted our exams to assess these desired concepts and skills.

The core of our assessment of student understanding and problem-solving skills occurs via course-wide exams administered over the span of two days. The first day is a traditional in-class exam during which the students do not have access to a calculator or computer. This portion of the exam focuses on basic fundamental concepts associated with the core mathematics program. Students are expected to develop mathematical models of real world situations. The focus of the exam has shifted from memorization to utilization and application of resources to problem solving. Upon completion of this portion, students are given a take-home, somewhat incomplete scenario that outlines a real-world problem. Students have the opportunity to explore the scenario on their own or in groups. Upon arrival in the classroom the next day, the final parameters of the scenario are provided, explored, and modified to allow students a chance to apply their problem-solving skills in a changing environment. Student growth is evident over the course of the semester as they improve their ability to apply learned concepts to new problems.

The current end state of our assessment is comprised of many different assessment instruments to provide a comprehensive picture of a student's learning. Each different assessment tool provides a different perspective. If it's true that "what you test is what you get," and if you never test skills without technology, then students neglect this. If you always use the same type of exam with only the subject matter changing, students become proficient at taking that type of test. As such, we have a fundamental skills test to evaluate proficiency in tasks we expect students to perform without the use of the textbook or technology. We have modified the remainder of the exams by allowing the students access to their textbooks. This is based on our belief that it is better for them to know where to go to get the right answer than for them to perform unnecessary memorization. Our exams, given in pieces, provide nearly equal weighting to skills with and without technology. Students without a strong conceptual understanding will not do well on the non-technology exam. Similarly, students without technological skills to solve computationally intensive problems will not do well on the technology exam. The term end exam is a similarly divided 3 ½ hour event requiring both non-technology and technology proficiency. The three projects given during the semester all contain different requirements. The first project is divided into distinct though interrelated pieces so as not to overwhelm the student with a large requirement. Together, these pieces combine to form a fully developed and analyzed project. As a part of this first project, student complete a portion in class as a one-problem exam. The second project involves a mathematically challenging problem whose end state is a two-page report summarizing findings. The final capstone project requires skills developed throughout the semester and culminates in an 8-10 page submission.

The notebook computer provides a tremendous resource for storage and organization of information. This resource creates the opportunity for students to transfer learning across time and between courses. The portable notebook computer provides the resource for students to create their own repository of learning. Creative exercises offer the student exposure to mathematical concepts with the ability to explore their properties and to determine patterns and connections which facilitate the process of constructive understanding. Thorough understanding is feasible in either a controlled learning environment or at the student's leisure. Instructors provide early guidance to incoming students on organizational strategies and file-naming protocol. Informal assessments of a student's electronic portfolio provide information regarding the ability to understand relationships between mathematical concepts.

Conclusion

As the world continues to change technologically, socially, politically, and economically, educational curricula must adapt and change to best prepare its graduates to meet the ever-growing demands placed upon our leaders and problem solvers. The undergraduate educational experience is a unique, sterilized microcosm designed to reinforce necessary fundamental and technological skills, problem solving processes, and critical thinking necessary for effective problem solvers. Continued refinements in what we teach, how we teach, and what we assess create the blueprint to focus student learning toward achieving our goals.

A Story of Success – and Failure – in an Applied Math Program Revision

Dr. Dale Peterson, USAFA,
Department of Mathematical Sciences

Introduction: A Civilian University Program and the Service Academies

For one and a half years I led an effort to revise the mathematics curriculum at a civilian university. The experience was at times exasperating, but in the end, eminently satisfying and extremely educational. I expect that those who embark upon a math curriculum revision will find the events related here to be of considerable interest and use. Among the issues we wrestled with were *breadth vs. depth* in the program, incorporation of *computing and modeling skills*, and *obtaining a consensus* among all department members.

The IUP program

The Department of Mathematics at Indiana University of Pennsylvania (IUP) offers an M.S. in Applied Mathematics. From Fall 1996 to Spring 1998 I chaired a committee to revise the program.

Although our program was a graduate program, it resembled many aspects of the programs here at USAFA in Mathematical Sciences and Operations Research (OR). The IUP M.S. degree was meant primarily to give graduates the skills to succeed in applied industrial work – and those skills are essentially identical to those needed by Air Force officers in analytical positions.

Curriculum Development Motivation

Several articles and reports from the mathematical sciences community recommend topics that are crucial for an applied mathematics/OR program – see [1, 2, 3, 4]. These articles consistently suggest similar topics. The 1996 AMS article lists these as (1) scientific computing, (2) modeling, (3) analysis, (4) probability and statistics, (5) discrete mathematics, (6) differential equations, and (7) optimization (OR). The utility of *modeling skills* is stressed throughout these articles.

My experiences with academic programs and industry

I was also strongly motivated by my own classroom experiences at Stanford, Brigham Young, and Rutgers Universities, where I earned my B.S., M.S., and Ph.D., respectively, and how they prepared me for my ten years of work in the aerospace field. In addition to courses in those topics listed above – notably numerical methods, OR, and probability/statistics – courses in *computer science*, such as programming and algorithm analysis, were essential in preparing me for industry. And, perhaps my single most useful applied course was one in *mathematical modeling* I took at BYU. The flexibility in the OR program at Rutgers allowed me to customize the program to my research interests in optimization and discrete mathematics, and I hoped to provide that same type of flexibility to my students.

Problems with the old curriculum – real and perceived?

Our master's program at IUP had no modeling, no simulation, no scientific computing, and little classical applied mathematics (analysis) – all topics overwhelmingly endorsed by the references. On the contrary, it was heavy in just two topics: OR and probability/statistics. Indeed, all five core courses were in these two areas. Moreover, as Ph.D.'s, the revision committee members were convinced it needed to provide flexibility in the program, from which we had all benefited in our Ph.D. educations.

Given these reports and my own experiences, and as chair of the revision committee, I was determined to mold the IUP curriculum into the "perfect" program.

Failure and Success in Changing the Curriculum

Prompted by the above reasons, faculty developed or revised several courses, all of which were approved by committee and then department votes[1]. These and existing courses addressed the crucial applied topics as well as including some pure mathematics. Except for the modeling course, all (or variations on them) were already offered as special topics classes. Our committee then crafted a curriculum change proposal, voted to endorse it, and passed it along for the department's vote.

Proposal 1 – Failure!

There were two major changes in our committee's proposal:

(1) *Reduce from five to four the list of core courses, balance it, and include modeling.* The new core would be comprised of four courses:

- Applied Mathematical Analysis
- OR I
- Mathematical Statistics I
- Modeling and Simulation

The first three courses addressed applied analysis, OR, and probability/statistics, respectively, and the fourth course incorporated elements from each of these three topics.

(2) *Revise and expand the list of electives so it included a better balance of topics.* By reducing the core, students could take six or more electives for credit. A student could obtain a broad-based education in applied math or could choose

[1] The revised courses were: Modeling & Simulation, Advanced Simulation, Applied Mathematical Analysis, Numerical Mathematics, Ordinary & Partial Differential Equations, Numerical Methods of Supercomputing, and Advanced Optimization. Existing courses included: Complex Variables, Graphs Networks and Combinatorics, Sampling Theory, Nonparametric Statistics, Applied Regression Analysis, Real Analysis I & II, Abstract Algebra I & II, Topology, Theory of Numbers, and Advanced Linear Algebra.

to emphasize probability/statistics, OR, or classical applied mathematics. The elective list included those applied math courses listed in the footnote; the pure math courses (analysis, algebra, topology, number theory, and advanced linear algebra) were dual level courses and were listed as additional electives.

Our committee was convinced that the proposal was excellent and that it would be approved by the department.

We did not expect the animated debate that ensued:

- One contingent argued that a single semester in probability/statistics was woefully inadequate; it could not provide the statistical depth to be of any real use.
- Another contingent argued that a single semester of OR deprived students of important elements of deterministic OR, stochastic OR, or both.
- An extremely important issue that arose was that, as a consequence of the flexibility, students would enroll in lower numbers in some courses – likely, including those we regard as crucial to applied mathematics. Indeed, we had observed this behavior in the past, and our committee had not considered that our proposal would exacerbate the problem. This might result in cancellation of courses and a loss of the very flexibility we sought.

The proposal went down – down hard, for those of us on the committee.

But in fact the department was almost certainly correct in disposing of our proposal. We would not be providing enough core depth to our students. We thought we were doing them a favor by providing flexibility. But, in fact, we know best what courses they need in order to be prepared for an industrial career. Immense flexibility is proper for a Ph.D. program, but for an undergraduate or master's program, the department probably had it right that we need to channel our students.

Proposal 2 – Success!

Our revised proposal actually *increased the core* to six courses, providing more depth:

- Applied Mathematical Analysis
- Deterministic OR
- Stochastic OR
- Mathematical Statistics I
- Mathematical Statistics II
- Modeling and Simulation

We maintained the list of electives from Proposal 1 (except for those we put into the core; in the meantime, the statistics committee had condensed its three elective courses into two), of which at least four could be taken for credit. *However, we "starred" three of these electives as highly recommended* for those entering industry:

- Numerical Mathematics
- Advanced Simulation
- Regression & Design of Experiments

We allowed four "controlled" electives: Foundations of Mathematics, History of Mathematics, Modern Geometries, and Special Topics.

The increased core meant a drop of two electives from Proposal 1 and put students on a similar track to make them general practitioners in problem-solving using applied mathematics. It also had the advantage of keeping enrollment up and evenly distributed in the graduate level courses. Students still had some flexibility, but not nearly as much as we intended in Proposal 1.

Proposal 2 was heartily approved by the department – *a culmination of hundreds of man-hours spread over a year and a half.*

Conclusions: The New Curriculum and Lessons Learned

Numerous reports and articles, and the IUP math department, agree that a *modeling course* is among the most useful that an applied mathematics program can provide. Modeling can incorporate several topics such as probability/statistics, OR, computing, differential equations, and classical applied mathematics – all of which are important elements in the program.

An important lesson learned is that we ought to *provide considerable structure* for students at this level who are preparing for industry. Industry typically requires breadth as well as some depth; attaining a reasonable level in both of these in an undergraduate or master's programs allows little room for error. Though some flexibility is desirable, extreme flexibility is perhaps best left for the Ph.D.'s who are attracted to academic and other positions where narrow specialization is rewarded.

Finally, this example illustrates as well as any I have experienced the advantage of *rule by committee and department*, with all of its varied inputs. Without the discussion and democratic process in the department, a handful of faculty – myself among them – may well have instituted a program less effective in the mathematical preparation of its graduates for industry, and may even have resulted in a damaged curriculum.

Acknowledgements

It was a pleasure to work with the members of the program revision department: Professors Rick Adkins, Dan Burkett, and Rebecca Stoudt. They devoted inordinate hours to this endeavor. I am also grateful to the then-department chair Jerry Buriok, whose service to the department contributed to an extremely supportive environment. Finally, I thank all the members of the IUP mathematics department, whose discussion and input resulted in the implementation of a solid graduate program.

Dedication

To the memory of Dr. Rebecca Stoudt, who was instrumental in the success of the graduate program, and who passed away in October 2001 from breast cancer. She was a good friend and colleague.

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Curricular Change – Some Lessons Learned

Prof. Brian Winkel, USMA, Department of Mathematical Sciences

One day a student in my mathematics class came to my office at Rose-Hulman Institute of Technology, where I was teaching applications of mathematics. He asked, "Do we have to use the math formulas for projectile motion? Why can't we use the physics formulas?" I said to him, "Do you think gravity is different over there?" He did not get it; ANY of it! So that night I went home and wrote a homework assignment entitled, "I Am Sick and Tired and I Can't Take It Any More!" I always titled my assignments, but never this negatively. The assignment was to get the physics text out and show how the little boxed formulae that physics authors package so well came from the development we had been offering in our class. Some of them got it, some did not. I was disturbed that my students were compartmentalizing the very things we were trying to relate; that they were learning inefficiently with little boxes for information instead of broad sweeping themes for success; and that they were not seeing the beauty of the mathematics I loved so well in action.

During the next academic year I applied for support and was funded by The Lilly Endowment to conduct a faculty development seminar with an outside visitor, Dr. H. T. Hanks, Applied Mathematics, Brown University. We gathered faculty to participate in an ongoing seminar about research in inverse problems and parameter estimation in modeling with differential equations. During our time together we also discussed our teaching, our concerns for our students' learning, and our own learning. From these discussions emerged a plan to investigate the possibility of doing something different, of coordinating curriculum efforts – indeed, of creating an integrated curriculum in first-year science, engineering, and mathematics. This effort is well documented.[2, 4, 5].

That one conversation with a student, those few gatherings of faculty, and many conversations with several of my dear friends at Rose-Hulman led to a massive effort to change the first-year curriculum in science, engineering and mathematics – ***an effort which eventually failed.***

What I want to discuss here is the process and how it could be changed and thus improved. Basically, there are a number of types of players in the complex roles of change agents – *visionaries*/*innovators*, *implementers*, and *doers*. There are also other figures in the drama – *positionaries*, *cautionaries*, and *resistors*. Visionaries and their kindred spirits – innovators – see things as they might be and want to move to that vision, while implementers attempt to bring about the change, to do the practical things necessary, and to smooth the way for happenings. Doers actually carry out the vision as implemented. In curriculum innovation the doers do; i.e., they teach and iterate the teaching model. Positionaries sit tight, enjoy the *status quo*, and may or may not be good at the *quo*; cautionaries sit on the side lines and caution, perhaps even chide, the others who would move on curriculum change; while resistors do just that – resist.

Player Awareness

Players who would change, who would move, need to be aware of others in the neighborhood: of their interests, of their attitudes, and of their needs. One need we all have is to have a sense of worth and belonging; and when change agents propose something different than the current strategy and move in that different direction, they need to be aware of the perceptions and reactions of others. Moreover, they need to be sensitive to others and they need to keep ALL dialogues open and fruitful.

Being a change agent is very difficult and few do it well; hence we do not see great changes often. Rather, we see gradual changes as a result of many smaller change agents and modest acceptors. Something new on the block is usually the result of several new things on blocks all around the neighborhood and some visitors from those blocks to our block – coupled with a local change agent or two, and a bit of luck and opportunity.

Benefiting from the Human Component

During the creative stages of curriculum development the visionaries or dreamers expend enormous amounts of energy on designing and creating the new curriculum – perhaps an all-consuming amount of effort, as was the case with the project in which I was involved. While this creating business is very hard to do, developers also need to spend time cultivating others to understand their vision. They need to seek and take ideas and suggestions from others to build a better, more acceptable product. Most importantly, they need to *listen* to colleagues.

I personally have been a dreamer who has designed curriculum, good curriculum – not just thought so by me, but others have said so. However, because our team did not benefit from the human component of our energy base – fellow faculty – we did not embrace their ideas from the start, nor did we fully involve them and give them ownership early in the development. It could be said that we alienated some of them – enough, in fact, that when the project could have been upscaled to the entire student body there was too much resistance. While we formed a faculty council to advise the curriculum effort, we understood its purpose very differently: they thought it was for them to give us their new ideas, and we thought it was for them to bless our ideas! So we just did not get it; we did not listen; we were not sensitive to the needs and the potential of others.

There were generic resistors, those that would not change. However, many were resisting because we just did not show them we cared about their ideas or their contributions. We were dreamers; we were naïve; we were novices in the game of politics of curriculum change and approval, and we could only keep track of a few things, albeit large things(!) – among them the development of a massive curriculum change effort as well as the assessment and evaluation of this effort. Often in the latter efforts we found ourselves attempting to counter our critics with data rather than embracing them in dialogue, thus alienating them even more at times.

In a paper to appear in the *Journal of Engineering Education*, four members of the Foundation Coalition (a broad engineering curriculum effort sponsored by the National Science Foundation) offer up "Evolving Models of Curriculur Change:

The Experience of the Foundation Coalition." Rose-Hulman Institute of Technology and the curriculum effort with which I was involved was a founding core of the Foundation Coalition. Several evolving models for curricular change are offered in the article with the fourth model being:

1. Develop the curriculum.
2. Pilot it and persuade colleagues to adopt it.
3. Implement it in a form that works for all students and faculty.
4. Devise structures and mechanisms to sustain its continuous growth.

There is a thread that is now understood by all involved. We affirmed this at a small conference of Foundation Coalition alumni at Banff National Park of Canada this summer. (We DO know where to meet!!) That thread is to solicit colleagues' ideas; to involve colleagues; to listen to colleagues' suggestions; to weave in the appropriate ideas to make the curriculum effort stronger; to permit others to take ownership; and to build a coalition of believers.

If I knew then what I know now, I believe I would have still proceeded. I would have incorporated teams to keep all involved more sensitive to the views, attitudes, and potential contributions of all about us as we worked to build a curriculum. Would the curriculum change we worked to build then still be here now? I am not sure. Of one thing I am sure: If anyone does begin a curriculum change and they do not involve colleagues from the start, do not listen, and do not become more inclusive of ideas and people, then that curriculum stands little chance of success. Listen to your intellect, your desire for change, your inner conviction; but also listen to others and bring them into the fold. You will be better for it, your curriculum change effort will be better for it, and your colleagues will be better for it.

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Enterprises of Great Pith and Moment

Dr. Lee Zhao, USMA, Department of Mathematical Sciences

It has often been a melancholy experience to hear a cadet ask: What is the point of a discrete dynamical system (DDS)? Of a determinant? Of a continuous function? I heard similar questions when I was teaching at Rutgers. Sometimes they were asked out of genuine curiosity, but more often they have been out of sheer frustration and disinterest. Indeed, there is nothing more disheartening than hearing skepticism about the value of your intellectual activities.

I can relate to this kind of frustration and disinterest. I often questioned the "point" of studying history during my naïve high school years. But I also remember that I *never* asked such questions in classes in which I excelled – regardless of the lack or abundance of the usefulness of the subjects. I only asked them in classes in which I performed poorly, as if the want of utility of the subjects (if any) would provide me with a justifiable excuse for failure.

I believe that when applied to mathematics, this kind of skepticism is unjust. Few of us ever question the "point" of the works of Chaucer or Shakespeare. Granted, their works have affected the development of the English language in very profound ways; but that is not why Shakespeare and Chaucer are remembered as great poets, nor is it why people make careers out of studying their works. They were great poets because they wrote great poetry and we study their works because of their aesthetic appeal. The importance and value of their works lie in the works themselves, not in any applications or consequences. Such is also true for mathematics. Both from purely academic and pedagogical points of view, the value of mathematics lies in itself.

Often I find the students to be disinterested in a subject and they perform accordingly poorly; what is more, their poor performances then compound their levels of disinterest. Thus a vicious cycle is formed and the skepticisms mentioned earlier are but manifestations of the disinterest. What I am disposed to write in this essay are some ideas I have had about the curriculum and things I have tried experimentally to break this cycle of disinterest.

I genuinely find mathematics to be interesting and beautiful. I cannot do better than quoting from Godfrey Harold Hardy [1] in this regard.

> *His [a mathematician's] subject is the most curious of all – there is none in which truth plays such odd pranks. It has the most elaborate and most fascinating technique, and gives unrivaled openings for the display of sheer professional skill.*

Indeed, mathematics has never failed in presenting marvels to its patrons. But somehow this interest is not felt by all my students. Hence I believed that it was one of my first duties as a college professor to impart some of this appreciation for mathematics to my students.

I was surprised to discover during the third summer of my graduate studies that Rutgers has a mathematics course entitled

Mathematics for the Liberal Arts. It is a freshman-level course; instead of having the students study the standard calculus or pre-calculus, it teaches elementary graph theory, probability, number theory, and combinatorics. It teaches concepts such as trees, Euler paths, modulo arithmetic, and the pigeon-hole principle. The course requires almost no pre-requisites whatsoever and can be taken by essentially anyone. It also touches upon problems such as decomposition of the Congressional districts and computing the consumer price index.

Relating this revelation to my fellow graduate students at the time, I found consensus among my peers that students would enjoy taking such a course much more than calculus or pre-calculus; I received similar feedback from the students. I felt that a course like that was genuinely much more interesting than any course in elementary university mathematics I have taught. All good mathematics has two characteristics: (1) *unexpectedness* as different ideas are connected; and (2) *inevitability* as conclusions show themselves to be inescapable. Elementary combinatorics, graph theory, and number theory often offer something that calculus and the like do not: economy. When problems are viewed from the right angle, one line of attack suffices to solve them. Indeed, we have all had the experience of seeing our students dread the sight of a lengthy calculus problem being solved on the board.

Pedagogically, I believe, and I would imagine that many people would agree with me, that mathematics is taught in universities because there are many crucial skills that an educated person should possess. Among these skills, perhaps the most important ones would be the abilities to reason and to present one's ideas in a clear and understandable fashion. No subject illustrates and imparts these skills to the students better than does mathematics. A calculus student will probably not remember the chain rule of differentiation years after his graduation. However, if teaching and learning have truly occurred, he will possess something much more valuable: the skills to solve complicated problems with reasoning and to present the solution clearly; i.e., *knowing how to make known the unknown*.

If those are our goals, then it matters not at all in what context we impart these skills. The reasoning that is required to show the infinitude of prime numbers or that the number of vertices in a tree is always one more than the number of its edges has the same pedagogical value as any problem such as computing the balance of an IRA using DDS or computing the volume of a irregularly shaped lake using a triple integral. Quoting again from [1]:

> *For what is useful above all* is technique*, and mathematical technique is taught mainly through pure mathematics.*

I am certainly not suggesting the changing of the calculus sequence at the college level, as such courses must be taken by all who study science and engineering. But rather, as I dread to see how many of my students have grown disgusted with the calculus, leading them to ultimately loathe mathematics altogether, I *am* suggesting that we should offer a course that teaches mathematics that would be considered interesting by students – the kind of mathematics that is understandable to all trained intelligence and that raises one's eyebrows upon sight. I believe that teaching elementary combinatorics, graph theory, and number theory to the liberal art students would offer precisely that. Such a course need not go deep into mathematics, but ought to cover many various topics. For a man of science or engineering, there is no escape from studying calculus; but he ought to be somewhat mathematically inclined in the first place. However, for someone who would benefit more richly by studying the spirit of mathematics rather than its content, perhaps calculus is not the only avenue for success.

The following is something I have done before. Many of my students are perhaps too concerned only with their grades to notice or appreciate the beauty of mathematics that is presented to them. There are also others who come to mathematics classes with negative attitudes, thinking mathematics is boring and useless and are simply taking the courses because they have to. The same can be said about many of the cadets at USMA I have taught. There is little I can do about this during class as there is much material to be covered. But to rectify the situation, I devised a few years ago the extra credit assignment of reading Hardy's *A Mathematician's Apology* [1] and writing a book report. Hardy's essay gives examples and philosophies of the beauty of mathematics perfectly, and it can be well-understood by non-mathematicians such as my students. More importantly, it contains two results in elementary mathematics that I believe most anyone would definitely consider interesting: Euclid's proof of the infinitude of the prime numbers, and Pythagoras' proof of the irrationality of $\sqrt{2}$. Neither requires more knowledge of mathematics than the Euclidean algorithm. My hope was that the assignment would generate some interest in mathematics in the minds of my students by showing them some of the most elegant results that our subject has to offer.

The outcome has been overwhelmingly positive. Not only has there been no extra-grading pain from reading the book reports, but I found much pleasure in the reading. I remember one of my students wrote to me after reading the *Apology*. He told me that after his college years, reading Hardy's *Apology* would likely be the only thing he remembers about his calculus course. Also there were students who expressed strong opinions of disagreement with Hardy and wrote passionately to that end. Many others agreed with Hardy and started to compare mathematics to poetry and the arts. One student enjoyed the experience so much that she bought copies of the *Apology* to share with her family members. This almost brought tears to my eyes. There is nothing more flattering to a craftsman than having him find many people interested in his trade. I was very pleased that many of my students read the essay and were indeed moved by it. Many of them did seem to have a better understanding of the beauty of mathematics about which they would otherwise be unaware. In brief, I was very happy with the outcome of this assignment and have been giving it to my classes ever since.

The mathematics in both the *Apology* and the Rutgers course are easily accessible by anyone. What is also true is that both provide the best that mathematics has to offer which is the best chance we have at such “enterprises of great pith and moment” ([2], *Hamlet, III, i, 88*): imparting the beauty of mathematics and generating interest in mathematics. For many of our students, we need to break the vicious cycle somewhere.

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[1] G. H. Hardy. *A Mathematician's Apology*, First Edition. Cambridge University Press, London, 1969, with a Foreword by C. P. Snow.

[2] W. Shakespeare. *The Norton Shakespeare*. W. W. Norton & Company, New York, 1997.

It is the Process – Not the Results

Prof. Thomas Lainis, USMA,
Department of Mathematical Sciences

Introduction and Motivation

Why change the curriculum? It has worked for a decade! Why should we teachers tamper with success? Two thoughts come to mind. The first is from the Greek philosopher Diogenes: "Nothing endures but change."[1] The second is a schematic of assessment (Figure 1) that was used in the Physics Department in 1995.

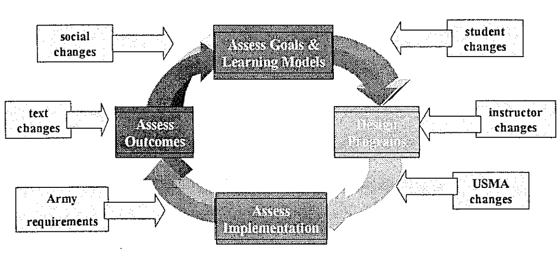

Fig 1: Education is a moving target.

If one studies Figure 1, one conclusion is inescapable: change is normal. Nothing affecting the curriculum is static. The curriculum must change – even if it is to stay the same! Perhaps this is exactly what Diogenes was thinking 2500 years ago.

At the U.S. Military Academy (USMA) at West Point, a major factor in initiating course changes has been the drive by the Academic Board beginning in 1995 to clearly articulate and define the goals of the Academy's Academic Program. Although West Point has had Academic Goals for years, it is only in the last decade that the various Academic Goal Committees have tried to describe in detail the implications of and assessment criteria for each of the goals. In addition, based on feedback from battalion commanders on what graduates can and cannot do well, there is renewed emphasis on both our interdisciplinary and problem-solving objectives. Interestingly, the observations by battalion commanders on our graduates mirror the results of a 1997[2] report on business leaders' identification of skills that all colleges need to emphasize for their graduates: problem-solving and working in interdisciplinary teams!

There is one final observation to be made. Colleges have an affinity for adding to *but not removing from* the curriculum. It is true that information and knowledge in the various fields have exploded in the last two decades. As an example, consider the time spent in graduate programs in physics. The average number of years in graduate school before graduation has increased from four to seven in the last two decades![3] At the

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[1] Heraclitus (540 BC - 480 BC), from Diogenes Laertius, Lives of Eminent Philosophers

[2] Department Heads Conference held at AAPT Headquarters, College Park, MD., in April 1997.

[3] Time to the Doctoral Degree at KU, An Analysis of Data from the Survey of Earned Doctorates, 1920-2000. http://www.ku.edu/~graduate/Pubs/GS_Report/Time_to_degree.shtml

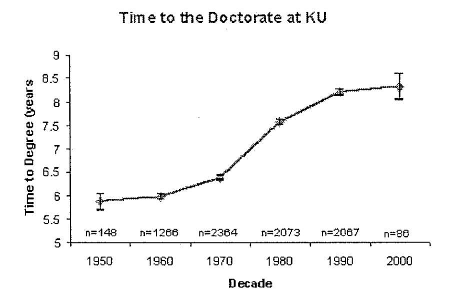

Fig 2: Degrees take longer.

undergraduate level, five year undergraduate programs are on the increase. The military academies do not have this luxury; Congress has stated that they are all four year institutions!

Math and Physics at USMA

The current programs in mathematics and physics can be traced back to approximately 1988 when USMA executed the current Academic Program. At that time, the three semesters of core physics for all MSE (Math-Science-Engineering) graduates were reduced to two semesters. In 2001, the Department of Physics initiated numerous changes in the core physics program by capitalizing on the results of the Introductory University Physics Program (IUPP). By combining a military storyline with a paring down of topics – emphasizing those physics principles most relevant to the use of Army equipment – the Physics Department is emphasizing depth of knowledge versus breadth of knowledge. In 1991, the Department of Mathematical Sciences helped lead broad changes in many school curricula by rewriting the mathematical science curriculum. It advocated a "7 (topics) into 4 (semesters)" program in which Discrete Dynamic Systems was incorporated into the curriculum. Not content to sit on their laurels, both Departments are examining ways to improve their courses. Math already emphasizes applications – ILAPs (Interdisciplinary Lively Applications) and MIPs (Modeling and Inquiry Problems) – but is looking for additional emphasis and relevancy by tailoring a new curriculum to correct student shortfalls in calculus plus developing a "better for the student" grouping of applications.

Technology

One recurring question is whether there is a means to increase the time spent understanding the complexities of problem solving versus the time spent on "mundane" mathematical computations. Many at West Point believe that through the use of technology and better curriculum coordination more efficient learning can be accomplished.

The use of technology to leverage learning can be divided into three broad subsets:

a. The WWW and Homework
b. Computational Expediency
c. Interactive Environments

WWW and Homework

Many instructors advocate the use of the WWW to increase student preparation (homework). At West Point, several departments are experimenting: Physics uses Web Assign; Chemistry uses Web CT; Math uses BCA (publisher provided program). The academy is migrating toward the use of Blackboard. How successful are these programs? It is still too early to tell. Perhaps the answer lies in the study by Bonham et al.[1] This study

[1] Bonham, Scott W., Dardorff, Duane, Beichner, Robert. "A Comparison of student performance using web and paper based homework in college-level physics" submitted to the *Journal of Research*

concluded that there is "no significant difference in student performance" between students encouraged to prepare using web-based programs for homework and those completing written homework requirements. This is the same conclusion reached by Dr. Evelyn Paterson, U.S. Air Force Academy, Department of Physics, in response to a question posed to her in an AAPT (American Association of Physics Teachers) presentation in 2001[1]. Dr. Paterson stated there was no difference in a comparison of results from several "standard" questions asked of students, but that she noted a difference in class discussions. Questions from the WWW-prepared students were always at a higher level!

Computational Expediency

Perhaps time can be saved by speeding up computations. Many departments are turning to computer software to reduce time spent in both seeing the situation or performing repetitive calculations. In Geography, 3D-map reading allows one to see the terrain as never before. In mathematics, this software was MathCad and is now (2002) *Mathematica*. Indeed, the software speeds up plotting (to see the situation or relationships) and algebraic manipulation. The issue appears to be how one introduces the computer to students such that the students do not think that the solutions are merely the results of "black box" computations. This is still under review. One thing is sure: the topic is both very controversial and emotional based on the lively arguments that have burst out at committee meeting between otherwise friendly colleagues!

Interactive Environments: Project Realms

Rose-Hulman Institute of Technology has been working with team teaching – especially during freshman year – for over a decade. In 1999, the physics and math departments at West Point offered a team-taught physics-math course. Statistically, the results were mixed. The instructors loved the course; student responses were somewhat diverse. There was an improved correlation between student grades in math and physics. Students were able to solve significantly more difficult problems. However, there were no significant increases in grades or levels of understanding that could be used to justify the increased time demands made on instructors in both departments. However, the appreciation of the students for using math in physics problems and physics in math problems did increase. How could we capitalize on this interdisciplinary factor?

There is another technology initiative being examined – Project Realms. Many instructors have noted that the current group of students have been raised on computer games. They are used to playing with the computer; i.e., make a move, receive feedback, and make another move. Visual and auditory input in these games are first class. The standard for academic usage of the computer is being set by Nintendo! Project Realms is a flexible 3D software template designed to leverage student desires to investigate, play, see, hear, and be "coerced" into studying. The students can play with inexpensive 3D software, or they can go straight to the homework. The homework explains

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<u>*in* Science Teaching</u> available on the WWW at http://physics.wku.edu/~bonham/Publications/publications.html

[1] AAPT National Conference in San Diego, August 2001.

requirements with links to the reading, tutorials, and questions uniting course work under a software umbrella. The bottom line is to grab the students in a single interactive program and keep them involved and motivated.

To add flexibility for instructors, responses are customizable; i.e., teachers can easily change the answers to better suit their perception of student needs or to assist with textbook compatibility. This program is still in its infancy.

Conclusion

Perhaps the question "What is the best curriculum?" has no answer.

Many instructors yearn to discover the Shangri-La of Teaching – that single combination of course material and teaching techniques that will make learning perfect for our students. We try to change one variable, such as the number of homework questions, thinking that all other variables remain constant and thus we will measure results with only small uncertainty. However, the other variables are not under our control – requirements from other courses, physical training demands, military demands, girlfriend and boyfriend issues, etc. These are never constant! There is no perfect curriculum. There will never be perfect and efficient learning. Learning and teaching are difficult activities.

This does not mean that we should stop looking. This does not mean that teachers should stop trying. Education research and curriculum changes based on sound epistemological principles should be made. However, the process is continuous. Teachers will always be able to continue to make improvements. The successes of this year may not be as effective next year.

The American Association of Physics Teachers conducted a study to identify those traits that best describe thriving and growing departments.[1] The conclusion was that there is no magic bullet. Rather, there were enthused faculty, supportive leadership, continuous evaluation, and support to teachers who take educated chances and who experiment with the curriculum.

I am continuously impressed by old and new instructors as they strive to improve teaching at West Point. In addition to the programs already briefly mentioned, there are faculty members trying to define and assess the fundamental mathematical skills of students; writing a calculus conceptual inventory; developing ILAPs and MIPs; co-teaching math & civil engineering, math & physics – even math & remedial studying. As teachers strive to achieve improvements, they know that this year's successful techniques will become tomorrow's obsolete techniques as vocabulary, student experiences, and societal demands all change. Teachers do not shy away from change, but thrive because of change. They search for student difficulties before they become student problems. They try to identify issues before they or the students become exasperated. The inspiration for these teachers may best be summarized by Henry Adams: "A teacher affects eternity. He

[1] Strategic Programs for Innovations in Undergraduate Physics Project available on the WWW, American Association of Physics Teachers, National Task Force on Undergraduate Physics, http://www.aapt.org/Projects/ntfup.cfm

can never tell where his influence stops."[1] At the military academies, we take pride in the knowledge that we are grooming tomorrow's leaders.

Still on the Bookshelf: Ritger and Rose *Thirty Five Years Later*

Fr. Gabriel B. Costa, USMA,
Department of Mathematical Sciences

Analysis has been the dominant branch of mathematics for 300 years, and differential equations are the heart of analysis.

George F. Simmons, *Differential Equations with Applications and Historical Notes*

Ever since I started learning about them, I've enjoyed differential equations: whether teaching a course on the subject, or working on a particular problem – like one I've been wrestling with for over fifteen years! Like all art, mathematics possesses the power to have a person "get out of one's self," thereby enabling one to feel refreshed and recreated. Put another way, I find differential equations to be *fun*. And I still have the book which first introduced me to the subject.

In 1968 the McGraw-Hill Book Company published a book on differential equations written by Paul D. Ritger and Nicholas J. Rose [1]. The text was part of the publisher's *International Series in Pure and Applied Mathematics*. Both gifted mathematicians spent many years teaching at Stevens Institute of Technology, earning well-deserved reputations as top-notch educators. Hence the text was a natural choice for use in the school's second-term, sophomore-year required course on differential equations.

As a nineteen-year-old mathematics major at Stevens, I remember even the jacket cover of the just-published text. Having already taken three semesters of calculus and a hybrid course entitled Matrices and Probability, I looked forward to learning about differential equations. From both my chemistry and physics courses, I had *heard* of them. But I didn't *know* what differential equations were … or how to *solve* them … or what they were *really used for*. I anticipated getting a lot of mileage out of the book.

The course required four weekly contact hours: two for lectures, delivered by a professor; and two for recitations, conducted by a graduate student. The lecturer's approach was *classical*: definitions, theorems (often with proofs), *analytical* solution techniques, and applications. Judicious substitutions and guesses were employed, as we chugged-and-plugged our way through the *standard* topics covered in such a course, finishing up with an introduction to partial differential equations. Any lingering questions were addressed during the recitation periods.

The text *was* put to a lot of good use. It was very readable and covered many areas. To this day, I still use the text. As I hear the binding crack a bit, I thumb through the now-yellowing pages … sections covering such things as Newton's Law of Cooling, Laplace Transforms, and Bessel's Equation are viewed. And I vividly recall studying these topics.

It is interesting to note, however, that some topics were not addressed. For example,

[1] Henry Adams, http://www.brainyquote.com/quotes/quotes/h/henrybada108018.html

the text's chapter on *numerical techniques* was never referred to in the course. I'm virtually certain that this was because the only existing "technical support" students had was a *log-log duplex deci trig* slide rule. Hence, anything beyond the most basic computations would simply take too much time. Likewise, there was no exposure to Euler's Method nor any introduction to *predictor-corrector* techniques.

While solution graphs and their properties are alluded to in the text, there is no real mention of *qualitative* methods. In fact, the word is not even in the book's index. As was the usual case for most mathematics texts, the black of the print was the only color seen on the white pages.

Things are obviously different today. Anything remotely resembling a CD-ROM, such as the *DETools* disk included with the text *Differential Equations* [2], would have been an unimaginable supplement for the Ritger and Rose text.

We have progressed.

As I look back, the Ritger and Rose text served its purpose exceedingly well. To this day, I still use it as a reference for myself and as a source for problems and exercises for my students. It provides a tangible link with my roots.

I also look ahead. The teaching of Differential Equations has greatly changed since the Ritger and Rose text made its debut. Interactive technology has given educators a myriad of tools. For example, the above-mentioned *DETools* disk provides multi-color graphs, phase portraits, and simulations which can be extremely helpful for today's educator. Properly used, technological advances can only help to better reveal the vast and far-reaching subject of differential equations. I believe Paul Ritger and Nick Rose would agree.

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[2] Paul Blanchard, Robert L. Devaney and Glen R. Hall, *Differential Equations* (2/E), Brooks/Cole, Pacific Grove, CA, 2002.