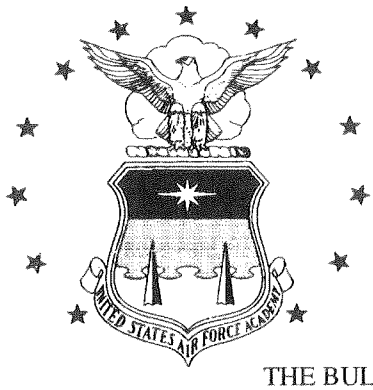
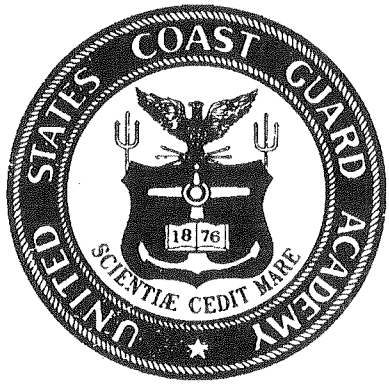
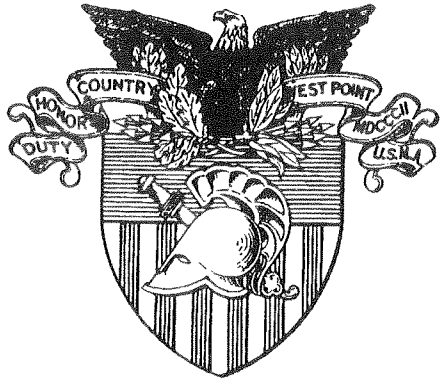


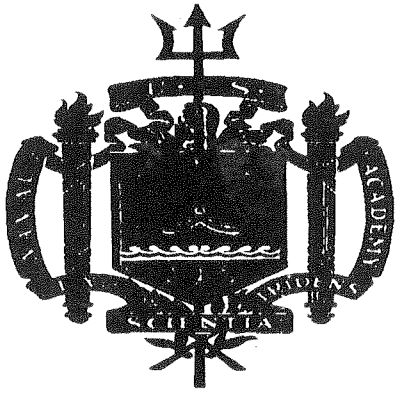
MATHEMATICA MILITARIS

THE BULLETIN OF THE MATHEMATICAL SCIENCES DEPARTMENTS
OF THE FEDERAL SERVICE ACADEMIES



FACULTY DEVELOPMENT AT THE SERVICE ACADEMIES





VOLUME 3, ISSUE 2
FALL, 1992

EDITOR'S NOTES

This issue features Faculty Development at the Service Academies . It also highlights the faculty research activities at each of the Service Academies. Our next issue will focus on the classroom environment.

The managing editor for the current year is MAJ Robert A. Rowlette, Jr. His phone number is A/V 688-2500, (914) 938-2500, and his e-mail address is rowlette@euler.usma.edu.

Mathematica Militaris exists because of the dedicated efforts of the editorial staff, particularly the associate editors at each of the academies. The time, effort, and creative abilities of our associate editors is greatly appreciated. We all hope that you, the reader, enjoy our publication and will provide your editor with any comments you may have to improve our bulletin.

We again express our gratitude towards the Association of Graduates, USMA, and **COLSA, Inc. Mr. Francisco Collazo**, President of COLSA, has underwritten the publication of *Mathematica Militaris* through a generous donation to the USMA Association of Graduates.

Best Wishes - Editorial Staff

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SUBSCRIPTIONS TO MATHEMATICA MILITARIS
IF YOU WOULD LIKE TO BE
ON OUR MAILING LIST,
PLEASE SEND YOUR NAME, ADDRESS, AND
AFFILIATION TO

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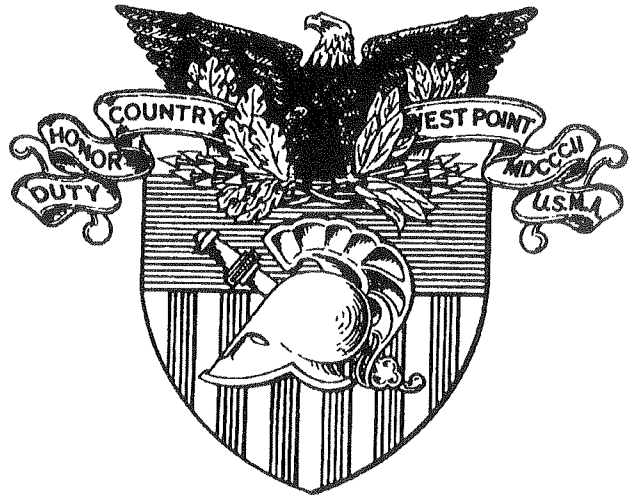
LEARNING TO TEACH DISCRETE DYNAMICAL SYSTEMS
A Faculty Development Workshop USMA Style



Each summer on the Monday after the 4th of July weekend, the Department of Mathematical Sciences at USMA begins its six-week Faculty Development Workshop (FDW1) for approximately fifteen newly assigned instructors with a picnic at COL Giordano's house. The purpose of this workshop is for these newly-educated instructors to learn how to teach Discrete Dynamical Systems, the course they will instruct beginning the 3rd week of August, and prepare themselves for teaching the rest of the core curriculum during the next three semesters.

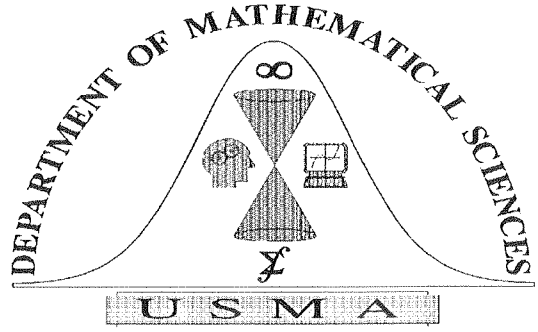
The major responsibility of the Department is to teach four core mathematics courses that each cadet takes in his or her first two years. In sequence these courses are Discrete Dynamical Systems, Calculus I & II, and Probability and Statistics. For the in-season semester, each of these courses requires at least 20 instructors. Discrete Dynamical Systems is in-season in the Fall; so, we use the FDW1 to prepare and have the new instructors teach this course first. These new instructors will then teach Calculus I, II and Probability and Statistics in sequence for the first two years. As a result of this faculty allocation model, the content focus of the FDW1 is teaching of Discrete Dynamical Systems.

The Department has been conducting this workshop in one form or another for over fifty years. At least 75% of the military faculty of the Department instruct at USMA for only three years and are called rotating faculty. To prepare these instructors, the Department sends these instructors to graduate school for two years to obtain masters degrees in operations research or applied mathematics. This takes care of most of the content preparation for teaching the four core courses that they will have to teach sometime during their three years. However, this does not prepare them to teach, and the Department fulfills this need with FDW1. Also, discrete dynamical systems are not commonly covered at most schools; so some content preparation in this course is necessary in FDW1.

After inprocessing in the Department and the picnic, the first week of the workshop is spent moving into quarters and into offices in the Department and getting an overview of the Department. Activities include a welcome from the Head of the Department, some briefings from the administrative staff, a briefing on the research opportunities the new instructors can expect, a two-phased seminar on teaching and learning, and briefings on each of the educational threads. The Department uses five threads to operationalize the student growth goals we have set for the four-course core curriculum. These educational threads are scientific computing, math modeling, history of math, communication in math, and math reasoning. These threads add continuity and structure to the curriculum and thereby give the instructor a good overall perspective of the curriculum.

The second week focuses on preparing the instructors with the tools of computation that they will use and an overview of the content of the four course curriculum. The sessions on the calculator and computer familiarize each instructor with their operation and the common software packages used by the students by hands-on exercises on their own machines. Since discrete dynamical systems is not a common subject in most graduate curriculums, we spend two days covering the content of this subject in depth. This past summer Professor Maynard Thompson from Indiana University gave two days of lectures on the topic. In addition, each of the other core courses are covered in a one to two hour briefing to give the instructor a good solid view of where we are going with our students in these two years of core math.

The third through fifth weeks focus on practice classes in lessons from the discrete dynamical systems course. The first two classes are presented by experienced instructors, and then in turn each instructor teaches two classes to a class made up of fellow new instructors. Following each of these classes, the instruction is critiqued by their peers in public and by an experienced instructor in private. In addition, each instructor has the option of having his or her class videotaped. These classes follow sequentially the significant lessons of discrete dynamical systems; so that by the end of the fifth week, the entire syllabus of this first semester has been covered. Also included is a briefing on student evaluation, and these new



instructors go through the first major test that the cadets take from how it is made up to actually grading their peers' work on the exam. Interspersed between classes are a tour of the library, a local history tour, briefings from the Dean's and the Superintendent's staffs, and a number of social events, to include a boat ride with the entire department on the Hudson River.

The final week of FDW1 is called Reorganization Week by the Academy. This week is spent integrating these new instructors into the department structure which will execute the Fall semester. The experienced instructors return from summer details and research projects to collect their thoughts and prepare for the new year. Specifically for the Faculty Development workshop, all that remains is a briefing from the Human Resource Council and the workshop after action review. The rest of the week is spent preparing classes and attending two course-wide meetings and four department-wide meetings. The week closes out with the "new instructors" losing to the "old instructors" in softball and the Department's annual square dance.

Since this workshop is over fifty years old, the Department thinks it is necessary and beneficial. And most of the feedback from the new instructors is that it is imperative. Edith Luchins from RPI, our visiting professor last year and a participant in FDW1 last year, was so impressed that she presented a paper at the joint MAA & AMS meetings in Baltimore conjecturing that all schools should have a similar program for new instructors. In addition, over the last two years the Department has expanded the workshop program and added three other workshops, FDW2 through FDW4, just prior to each of the other core courses and a final workshop, FDW5, given in the Spring to second year instructors who will assume course leadership roles the next academic year. These workshops are much shorter and content specific.

Only FDW1 teaches the details of how to teach, which seems to require the longer experience.

by LTC R. West

FACULTY DEVELOPMENT AT USAFA

Since the Department of Mathematical Sciences is almost exclusively a military faculty, our turnover rate is typically 25% each year. With 8-10 brand-new instructors per year, and with the average experience level around 3.5 years, we aggressively pursue faculty development. The formal New Instructor Training Program lasts four weeks. Through panel discussions, lectures, and tours, new arrivals are introduced to the department's facilities and operating procedures. They also receive numerous hours of instructor training to include three controlled lesson presentations, with experienced faculty and the other new instructors acting as students. The second lesson is recorded on video, which the new instructor can review.

The Dean of Faculty also concurrently provides an orientation program focusing on the mission and structure of the Academy and interdepartmental items of interest. Specifically, new instructors are introduced to the Academy's honor program and military training philosophy. Workshops are also held to discuss educational topics like cooperative learning, writing across the curriculum, and educational technology.

In the spirit of continuous instructor improvement, the department offers periodic "teacher tales" seminars, examining various aspects of instruction, classroom discipline, test design, etc. Experienced instructors serve as small group facilitators, and free discussion of the topic is encouraged. Summaries are recorded for a reference book that non-attendees can use. The department also has a vigorous formal observation program that includes no-notice teacher evaluations by the senior staff.

Next year, in response to feedback by previous classes, we are extending the "mentor" program that we use during New Instructor Training . Each new instructor will be assigned a mentor, an experienced teacher who will serve as a friendly technical expert and non-judging classroom observer throughout the academic year.

There has been a growing recognition of a need to consolidate all our faculty development activities under a senior member of the department so that they can receive an appropriate focus. For this reason, the Department Head recently created the senior staff position of Deputy Head for Faculty Development.

by Lt Col Bill Kiele

UNITED STATES AIR FORCE ACADEMY
DEPARTMENT OF MATHEMATICAL SCIENCES
GUIDELINES FOR CAREER PROGRESSION

FIRST YEAR

SUMMER
- Complete New Instructor Training Program

FALL
- Full teaching load, usually consisting of five sections of freshman core mathematics
- Look for a cadet military training program to participate in next summer
- Select research or mission support "track" (or some of both)
  — If research, consider ongoing research projects in the department
  — If mission support, make contacts to get involved in cadet military training or athletics
- If rated, start flying in support of cadet flight training programs

SPRING
- Full teaching load, usually consisting of five sections of freshman core mathematics
- If research track, start on a project
- Select a significant additional duty (like Honor Liaison Officer or Assistant Personnel Officer)
- Make future teaching and course directing desires known to supervisor and appropriate academic division chief
- If interested in PhD sponsorship, let supervisor and department head know

SECOND YEAR

SUMMER
- Teach a 3-week summer academic period
- Participate in a cadet military training program like Basic Cadet Training

FALL
- Full teaching load, usually consisting of five sections of core mathematics
- Possible download to three or four sections for course directing, research, mission support, flying, or significant additional duties
- Actively seek a "home" division (like statistics, applied math, or analysis) and course directorship

SPRING
- Teach in home division with possible split teaching load in core
- Start thinking about next Air Force assignment

THIRD YEAR

SUMMER
- Officers selected for PhD sponsorship depart
- Participate in two summer activities (like new instructor training, course development, flying, administering placement exams, or research)

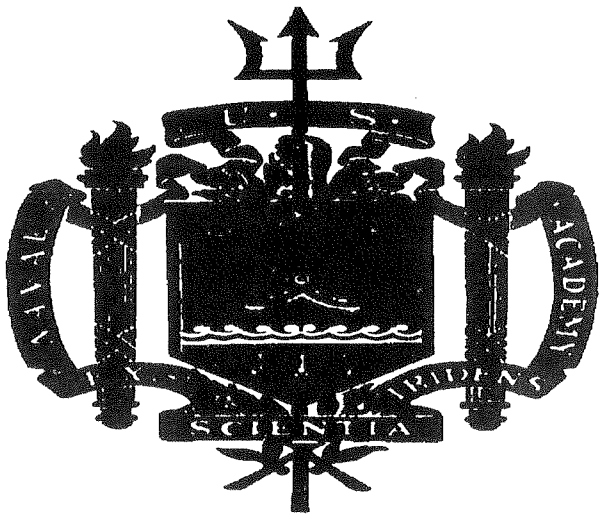
FALL
- Reduced teaching load to two, three, or four sections for most instructors due to course directing, research, mission support, flying, or significant additional duties

SPRING
- Settled in home division
- Investigate next Air Force assignment options

FOURTH YEAR

SUMMER, FALL and SPRING
- Same as last year
- Actively seek next Air Force assignment

by Col Daniel W Litwhiler



FACULTY DEVELOPMENT at the USNA

Recognizing the importance of a vital and up to date faculty in the education of Naval Officers, the USNA provides a sound basis of support for development of faculty expertise in the areas of research, teaching and extra curricular activities. We shall focus on the traditional areas of research and teaching. New faculty are normally supported for up to three years by the Office of Naval Research through funds administered internally by the Naval Academy Research Council. This gives junior faculty members time to establish their academic credentials and seek external support through agencies such a ONR and Navy and Federal Laboratories. Successful collaborations are then maintained on an annual basis providing continuity in support and research activities. Alternate means of support exist. The Academic Dean & Provost provides limited, select summer support for high quality faculty research which has not been funded by other sources. The departments may provide some travel support. Mini-grants are available from the Faculty Development Committee supported by the USNA Alumni to cover travel for special events and lectures, outside speakers, page charges and other items of a unique or "one time only" nature. These could be seminars, special workshops held in conjunction with conferences or an international congress at which a faculty member is speaking.

The Mathematics Department has on going seminars in the areas of Cryptology and Parallel Processing. These provide a weekly forum for discussion of projects and a training vehicle for faculty interested in broadening their education. Moreover, the Department has both an Applied Mathematics Seminar and Colloquium. These meet twice weekly and bring in speakers from other departments in the Yard and other institutions. From time to time, various groups have found funding support for a research professor. One such example is the Operations Analysis Research Group which through external grants brought in three researchers over a four year period to support faculty and midshipmen-faculty research projects.

The Naval Academy Instructional Development Advisory Committee recommends funding for course development through the Academic Dean and Provost. These grants support faculty efforts to create new and innovative educational tools custom tailored to midshipmen needs and interests. One such effort has lead to the development of a software package called "MPP" or Midshipmen Plotting Package which has proved a useful tool in graphing data arising in the calculus and differential equations courses at the Naval Academy. The MPP, available as free-ware, has been adopted at many colleges and high schools. The software package "MDEP", short for midshipmen Differential Equations Package, is widely used for numerical and graphical solution displays in the core differential equations courses. Another example is a teaching package for numerical methods and analysis which is directed toward upper level mathematics course taught in the Turbo PASCAL environment. The results were course materials, notes, numerical algorithm code and software, and graphics support.

The Naval Academy enjoys what is probably the finest undergraduate mathematics library collection in existence. The collection of books and journals at Nimitz Library is comprehensive and up to date. Various services are provided through interlibrary loans and on-line data bases. A highly competent reference staff works effectively with faculty and midshipmen in meeting research and instructional requirements.

MATHEMATICAL RESEARCH AT USAFA

Thirty-one officers in the Department of Mathematical Sciences at the Air Force Academy were involved in research during the 91-92 academic year. While over 50 research topics were investigated, four of the strongest efforts are highlighted below.

Calculus Reform
Lt Col William Kiele

For the past two years, calculus reform at the Air Force Academy has focused on developing in our students better problem-solving and cognitive skills. The reduced emphasis on algebraic, rote-memory skills has been made possible by integrating computers into the curriculum. We discovered that it took almost a full year of exposure to our reformed calculus before students displayed measurable improvements in their cognitive skills when compared to the traditionally taught control group. We also found that an emphasis on improving student comprehension of technical reading material must accompany any shift away from skills-based calculus courses. Simply stated, most freshmen can't or won't carefully read technical material without guidance. A preliminary report detailing these and other findings is available, and a formal technical report will be completed later this year.

Design of Experiments Short Course
Lt Col Ron Berdine

The Department of Mathematical Sciences has offered a short course in Design of Experiments to various DOD agencies for over three years. It was developed from a Total Quality Management perspective with emphasis on process modeling and improvement. In the past, experimentation has been conducted using a "full-factorial" approach which is usually too expensive and time consuming, or a "one-at-a-time" approach which does not allow adequate model-building. The focus of this course has been to provide modern tools to managers, engineers, and analysts responsible for conducting experiments, which will result in increased information with fewer experimental runs. We cover classical ANOVA, full and fractional factorial designs, Taguchi methods, regression, and variance reduction techniques. Specific designs such as Box-Wilson, Plackett-Burman, Latin Squares, Box-Behnken, and Foldovers are covered in detail.

War Game Geometry
Capt Larry Chilton

The Air Force Academy is developing a computer war game using laser disc technology for all Professional Military Studies (PMS) courses. Other DOD agencies, including the Army War College, have expressed interest in the game. One of the design principles of the game is flexibility. The game provides the capability to study battles from the past, as well as look to future battlefields such as space. The first phase of the mathematical problem involves determining areas of intersection of pairs of ellipses in the plane. The second phase involves determining volumes of intersection of pairs of ellipsoids and other quadric surfaces in space. The first phase is finished and the game is scheduled to be used in PMS courses beginning Fall 1992. Phase two is underway and will be added to the game when complete.

Testbank
Capt Francine Lockwood

Testbank is an automated test development data base program designed to enhance the reliability and efficiency of test development. *Testbank* accomplishes these goals by maintaining a data base of test items together with reference data for each time the item is used. A test developer can query the test data base for items that satisfy any number of parameters. A full range of test editing functions are available to modify and revise the test. The reference data for the individual test questions are used to predict the mean and standard deviation for the entire test. We currently have five courses with ongoing test data bases established. One of the many benefits of *Testbank* is the ability to confidently implement criteria-based grading schemes, as opposed to "grading on the curve."

MATH-SCIENCE FACULTY RESEARCH PROJECTS at the USNA

APPLICATIONS of ARITIFICAL INTELLIGENCE and GRAPH THEORY

Associate Professor Carol G. Crawford

Dr. Carol Crawford's principle research is a collaborative effort with Dr. Eric Mjolsnes, Yale University Computer Science Center, which concerns the development of innovative mathematical methods in artificial intelligence for the FBI's automated fingerprint identification. The researchers studied techniques utilizing relaxation algorithms, Zucker's splines, elastic nets, smooth local symmetries and graph matching neural nets to develop methods that were applied to specific identification problems. The techniques included image processing, trainability, scalability and robustness. Forth coming investigations will be in cooperation with Yale University's newly opened Center for Theoretical and Applied Neural Science.

Dr. Crawford is a versatile researcher who also continues a cooperative project with Dr. Stuart Ullman, David Taylor Research Center, on the development of a method for utilizing properties of graph colorings for mapping task graphs to parallel processing networks. The idea is to map the modules of a task graph to the processing elements of a parallel computer in order to minimize the total execution complexity of a task. The focus is on a special class of problems where the task precedence graph is almost a full tree and the parallel processor is connected as a hypercube.

LOGIC, GROUP ACTIONS on TREES and MODELS

Professor Anthony M. Gaglione

Dr. Gaglione's research concerns connections between the universal theory of free groups and group actions on Lambda-trees where Lambda is an ordered Abelian group. The researcher develops properties of these structures which allow him to prove theorems of the following type: "Every non-Abelian group having the same universal theory as the non-Abelian free group is tree-free." Moreover, by allowing vacuous quantifications, a conjecture arises which is the subject of continuing investigation. It is: "The class of non-Abelian tree-free groups is precisely the model class of those universal and existential sentences of first-order group theory true in every non-Abelian free group."

GRAPH THEORETIC PROPERTIES

Assistant Professor T.S. Michael

Recently, an open problem arose as to whether or not the complete graph of ten vertices admits a decomposition into three copies of the Petersen graph. This is answered in the negative. The proof was based on a clever linear algebra argument that showed that such a decomposition was impossible. Dr. Michael has shown that the complete graph on "n" vertices can be decomposed into three copies of a strongly regular graph only under certain highly restrictive conditions on the parameters. Dr. Michael also introduces a structure matrix into the study of the class of multigraphs with a prescribed degree sequence "D." With the structure matrix he obtains necessary and sufficient conditions for the existence of a graph with degree sequence D. And, a formula for the maximum size of a matching among all graphs with degree sequence D is provided. The study of graph theory is of particular interest to theoretical chemists and mathematicians.

Faculty Research Supports Army Missions

This past year marked another productive one for the officers of West Point's Department of Mathematical Sciences. The Department encourages all its military faculty to engage in a vigorous research program, and, when possible, to tie that research to the needs of an Army laboratory or technical agency. What follows is a sample of the type of work accomplished during the previous academic year:

Colonel John Edwards, Captain Philip Schlatter, and Captain Buck Buchanan participated in the 1992 Combat Identification Test at White Sands Missile Range during the summer. These officers helped design the test plan and evaluate information obtained during the test. Colonel Edwards provided additional extensive technical advice and support to the Combat Identification Systems Project Office throughout the entire year.

Colonel Edwards, Captain Dan Hogan, and Captain Hugh Bell conducted special studies for the Commandant of the U.S. Army Air Defense School.

Captain Edward Healy performed design studies of improved helicopter airframes at the NASA Langley Research Center during the summer.

Captain Steve Horton performed basic research in graph theory together with members of the faculty of Georgia Tech.

Lieutenant Colonel Jack Robertson provided support to the Center for Electronic Warfare Reconnaissance, Surveillance and Target Acquisition during the testing and evaluation of the Non-Imaging Sensor for Non-cooperative Target Recognition field test. He also continued to function as a member of the field staff for the Signatures, Sensors, and Signal Processing Technology Office, U.S. Army Laboratory Command. Lieutenant Colonel Robertson also presented two papers—one on acoustic modeling and the other on chaos—to the 10th Army Conference on Applied Mathematics and Computing, held at West Point in the late spring.

For further information on these and other research activities, contact LTC Robertson, the Director of Faculty Development and Research at DSN 688-2453 or Comm (914) 938-2453.

Prof. Devaney Visits USMA

Professor Robert L. Devaney, the former Chairman of the Department of Mathematics at Boston University and a leading researcher and author on dynamical systems, addressed the USMA Class of 1996 and cadets from other mathematics and physics courses on 29 October in South Auditorium of Thayer Hall. During the lecture, Professor Devaney discussed nonlinear dynamical systems, particularly ones involving quadratic functions, and he showed the audience a detailed exploration of the Orbit Diagram and Mandelbrot Set, two of the most intricate and beautiful fractals from all of mathematics.

The material presented in the lecture is an extension of the Discrete Dynamical Systems mathematics course taken by the class of 1996, and showed the cadets another aspect of the material they are currently studying, especially non-linear equations. The applications to other courses was evident in the complexity that arises from the analysis of simple nonlinear systems, which has given rise in the past fifteen years to a new branch of mathematics known as Chaos. Next year, the Department of Mathematical Sciences will offer for the first time a course on Chaos, which will allow cadets to further investigate the material presented by Professor Devaney.

Professor Devaney's lecture was an exciting multi-media presentation, which included short film clips on the Mandelbrot Set which Professor Devaney developed under a grant from the National Science Foundation, a demonstration of the dynamical systems computer package developed at Boston University to highlight the intricacy of chaotic systems, and several color slides which allowed the cadets to examine the true detail of these dynamical systems.

Following the lecture, Professor Devaney presented a seminar to the faculty of the Department of Mathematical Sciences, during which he discussed research topics which are still open in the field of Chaos. He also explained how he has integrated this material into the core mathematics curriculum at Boston University and discussed his three textbooks on the subject.

The Distinguished Lecture Series is made possible from a generous grant from the USMA Class of 1951. The last mathematician to give a lecture in this series was John Allen Paulos, who presented concepts from his popular book , Innumeracy.

by CPT P. Beaver

NAVAL ACADEMY MIDSHIPMEN PRESENTATIONS SASMC 1992

The second annual Service Academy Student Math Conference was held at the Math/Science Complex at Chauvenet and Michelson Halls, USNA from 3-5 April. The conference provided an excellent opportunity for midshipmen and cadets to discuss research and independent reading projects. Twenty minute presentations by midshipmen 1/C were based on independent projects. The abstracts are listed with the names of the midshipmen and faculty supervisors.

Discrete Wavelet Analysis with Applications in Radar Processing (MID 1/C Paul Wilson Miller & ASSOC PROF David W. Joyner)

Abstract: We define the discrete wavelet transform and discuss the wavelet decomposition of a discrete function. The Haar wavelet is given as a simple example and some applications are briefly discussed. (Assoc Ed remarks: "Wavelets" is a rapidly developing branch of mathematics similar to Fourier analysis. The wavelets are computationally simple, flexible and easy to "customized" to a specific problem. The wavelet transform partitions data into different frequency components and studies each component with a resolution matched to its scale. Navy interest is in radar data processing for the SLAM Missle. For a description of an ONR supported Accelerated Research Initiative see "Business Week" magazine.

Continued Fractions (MID 1/C Carl Harvey Ebersole & LT Christopher Sagovac)

Abstract: Continued Fractions are developed using an algebraic method as well as using the Continued Fraction Algorithm. Convergence of Continued Fractions is defined and sufficient conditions for convergence are stated. Euler's transformation, which transforms a power series into a continued fraction having the same radius of convergence and speed of convergence, is stated and an example given. Gauss' Continued Fraction Expansion is stated and an example is given, following the definition of confluent hypergeometric functions. A problem appearing in a recent math journal is solved. The problem is as follows: Given two successive denominators of partial convergents of the continued fraction for a number between 0 and 1, find the numerators for those same two successive partial convergents.

All midshipmen are required to complete a "capstone" paper in their major. This year many of the math majors chose one of "Hilbert's 23 Problems" presented by Professor David Hilbert at his plenary address to the International Congress of Mathematicians in 1900. Midshipmen's capstone papers reviewed by Prof Mark Meyerson were given as two minute presentations.They are listed below by title and author. The problem number in Hilbert's original list is designated [H(number].

[H1] "The Continuum Hypothesis," Jason Jack

[H2] "The consistency of the axioms of arithmetic,"Heidi Althoff

[H5] "How far does Lie group theory go?" Jeffrey Draeger

[H7] "Exponents and transcendental numbers," Thomas Schmidt

[H8] "Distribution of primes," Laurence Young

[H9] "General law of reciprocity," Les Fiest

[H10] "Diophantine equations," Eric Brenden

[H12] "Roots of polynomials in field extensions," Kirk Loftus

[H14] "Are certain rings, such as rings of invariants, finitely generated?" Stacy Marcott

[H16] "Branches of algebraic curves," Jeff Todd

[H17] "Positive definite rational functions," Katherine Badgley

[H18] "Tiling with a fundamental region,tiling with a non- fundamental polygon and densest sphere packing, "Kurt Zahnen (talk by Tanya Mayer)

[H19] "The analyticity of regular problems," Susanne Franklin

[H20] "Solvable bounded variation problems," Derek Wessman

[H21] "The existence of linear differential equations with prescribed monodromy, Scott Conway

[H22] "Uniformization," W. Conrad Fuller

For additional reading on Hilbert's Problems see Hilbert by Constance Reid. This is an excellent biography, now out of print, but available as a pair of biographies Hilbert-Courant by C. Reid, 1986, Springer-Verlag. Also, see Mathematical Developments Arising from Hilbert's Problems, Proceedings of Symposia in Pure Mathematics, edited by Felix Browder, 1986, American Mathematical Society.