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Resilience Analysis of Various Communication Network Models

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motivation and key supporting concepts

INTRODUCTION

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Motivation

Military planners, information technology specialists, and many other professions benefit from insights afforded from network models. There exist many ways to model a network, usually influenced by design choices such as:

- keep it simple
- capture every detail imaginable
- ignore those and focus on these

These, and other considerations, may result in different models of the same fundamental network. This begs the question:

What are the unintended consequences of choosing one model over another?

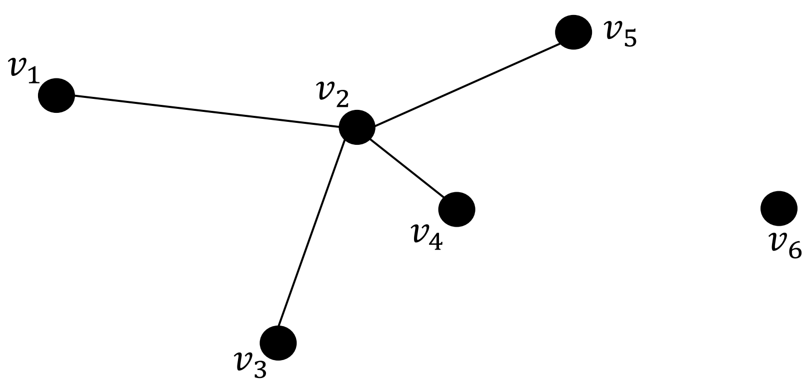
Concepts

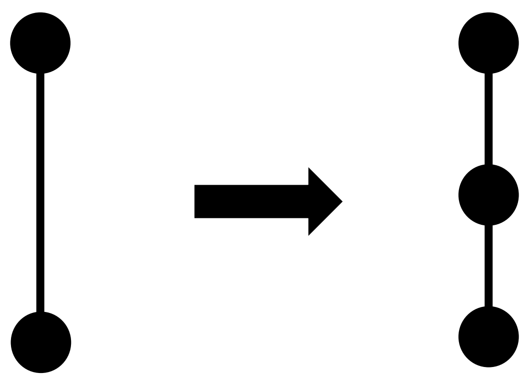
<u>subgraph</u> – given G, a graph H wherein $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

<u>component</u> – a subgraph H of G wherein $V(H) \subset V(G)$ and $E(H) \subset E(G)$.

<u>betweenness centrality</u> – for node i, an associated value $x_i = \sum_{u,v \in V(G)} \frac{n_{uv}^i}{g_{uv}}$ where g_{uv} is the number of uv-geodesics and n_{uv}^i the number of uv-geodesics containing i.

<u>edge subdivision</u> – given some $uv \in E(G)$, the process of adding a new node w to $V(G)$, removing uv from $E(G)$, and adding uw and vw to $E(G)$.





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modeling methods and final networks

NETWORK MODELS

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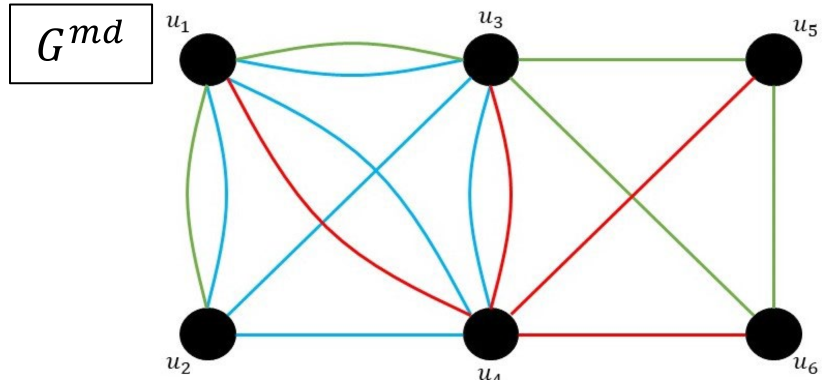
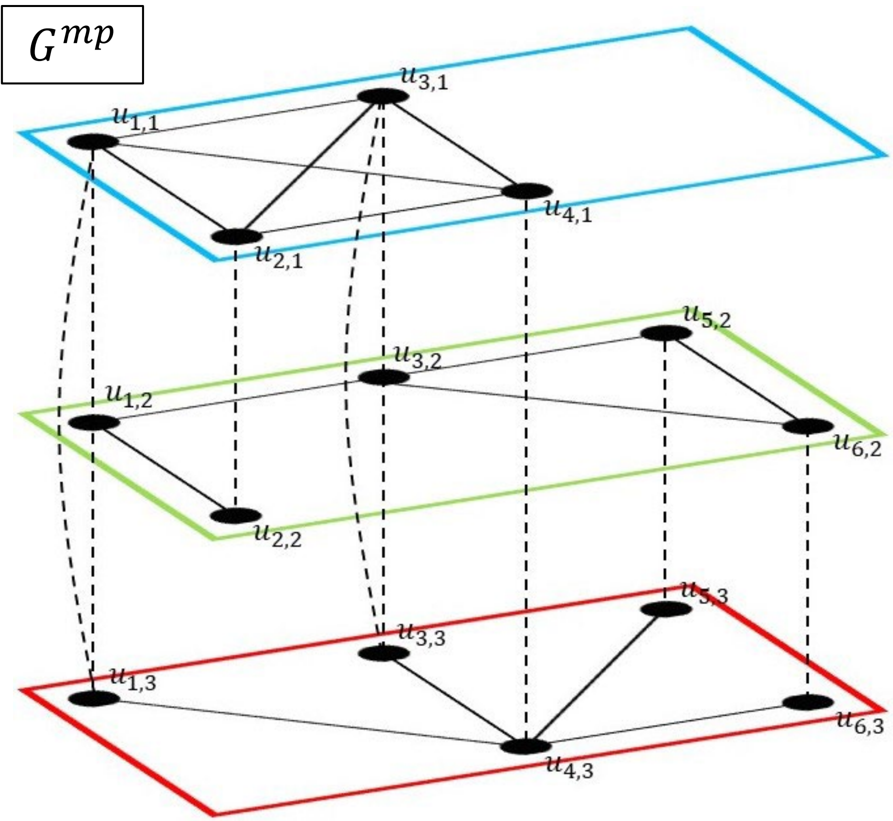
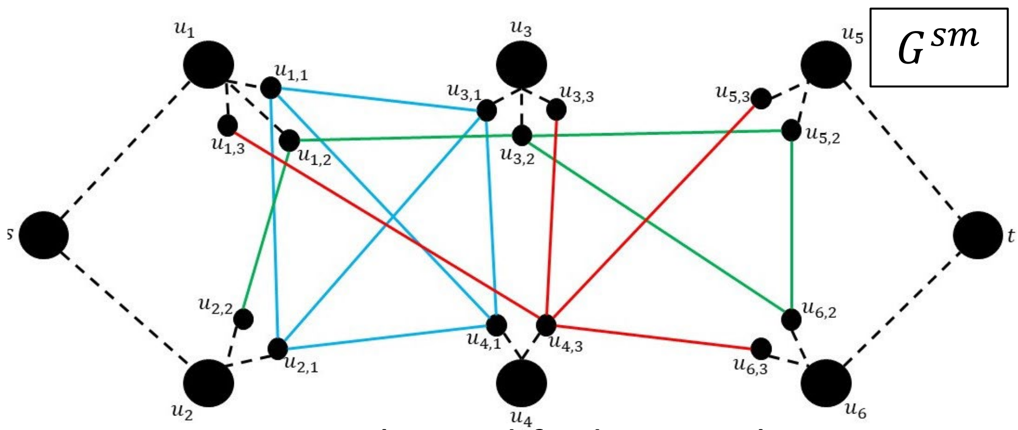
Modeling Methods



Multidimensional network



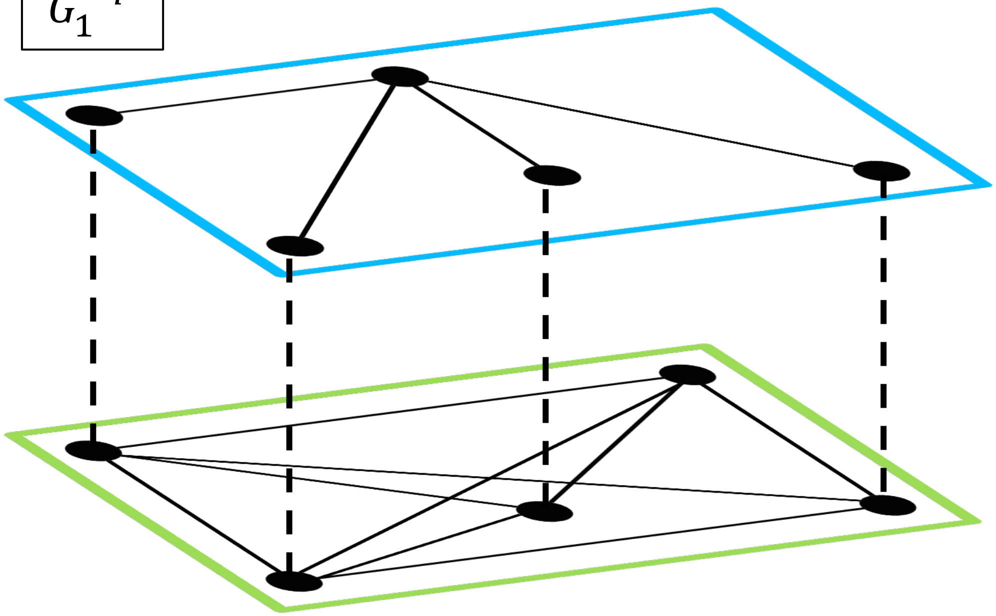
Multiplex network



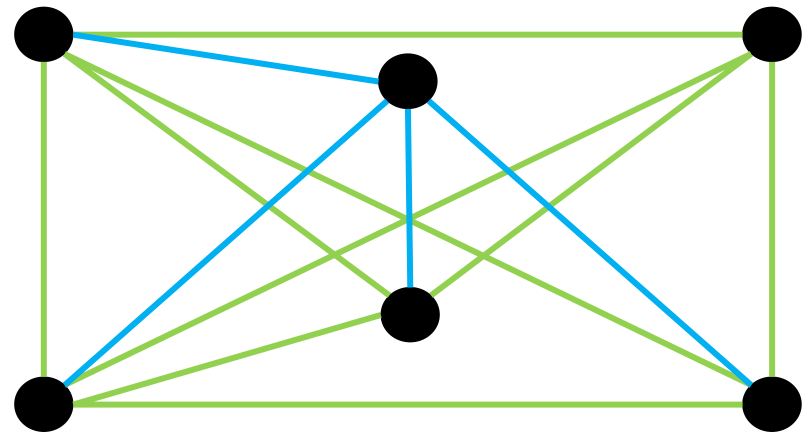
Simple-modified network

Networks (G_1)

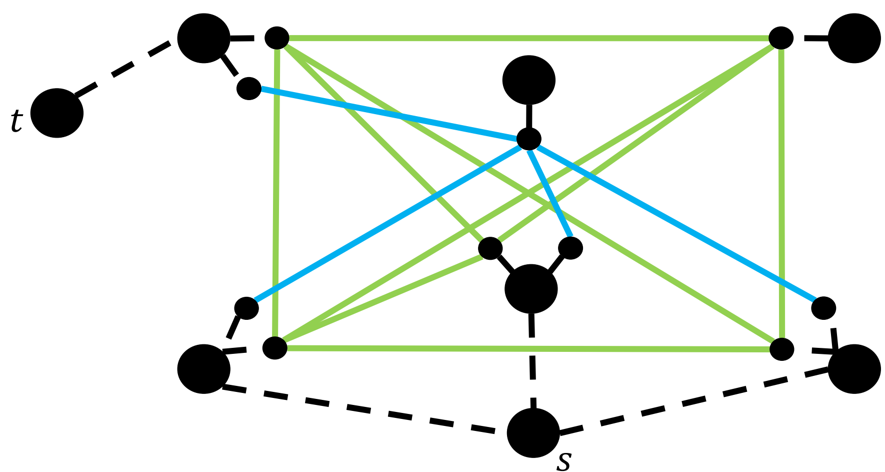
G_1^{mp}



G_1^{md}



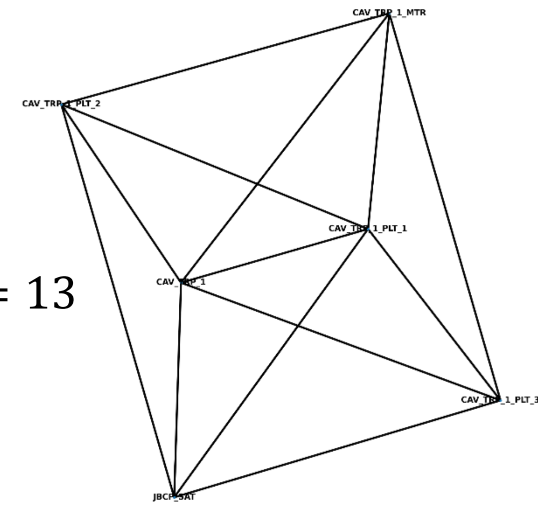
G_1^{sm}



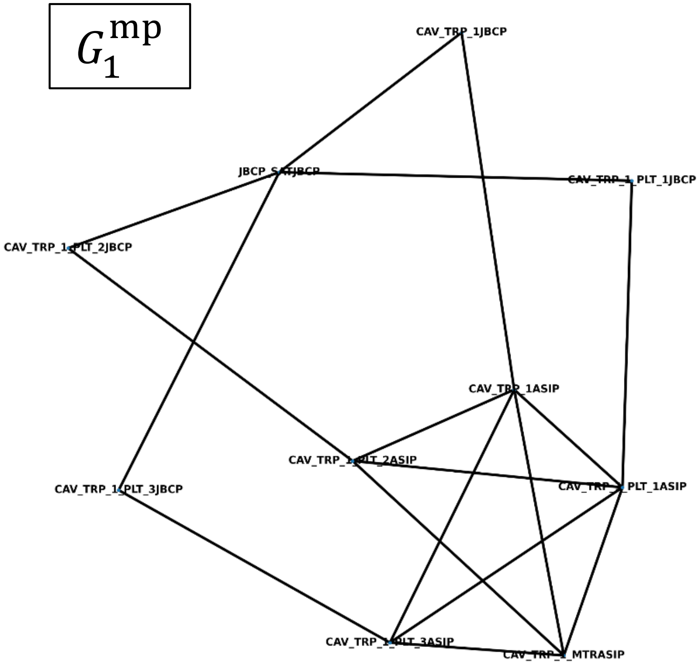
Networks (G_1)

G_1^{md}

$n = 6, m = 13$

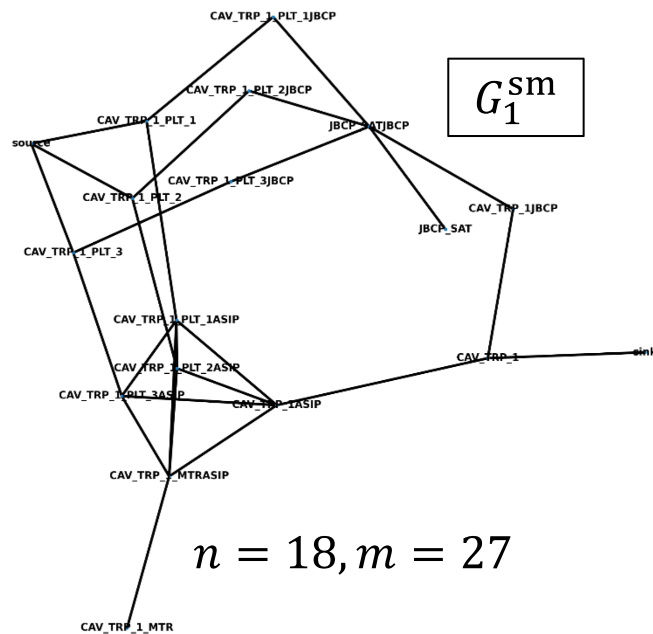


G_1^{mp}



$n = 10, m = 17$

G_1^{sm}



$n = 18, m = 27$

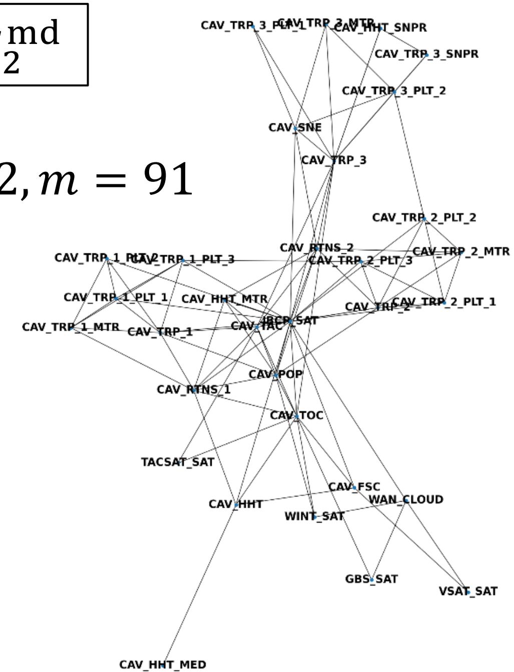
Networks (G_2)

G_2^{mp}

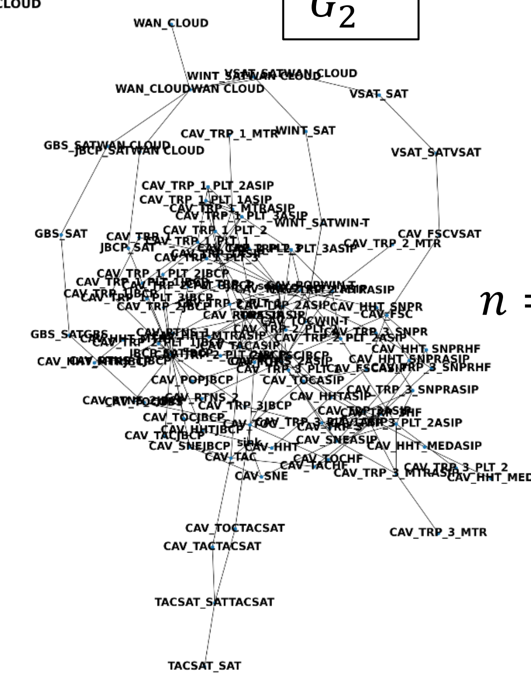
$n = 66, m = 142$

G_2^{md}

$n = 32, m = 91$



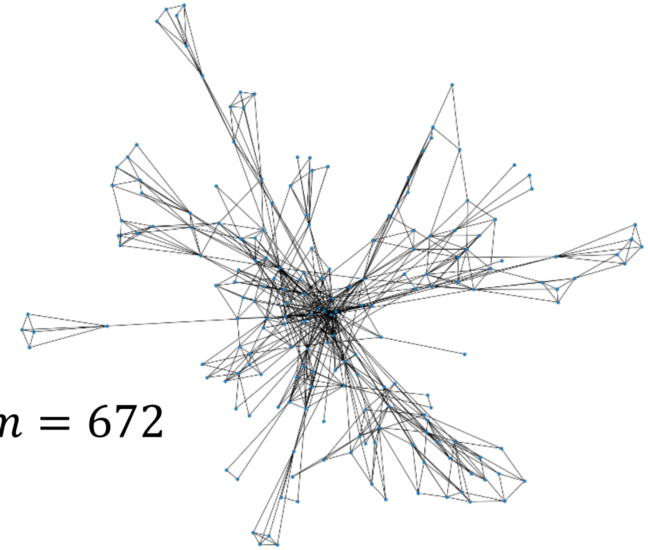
G_2^{sm}



$n = 99, m = 170$

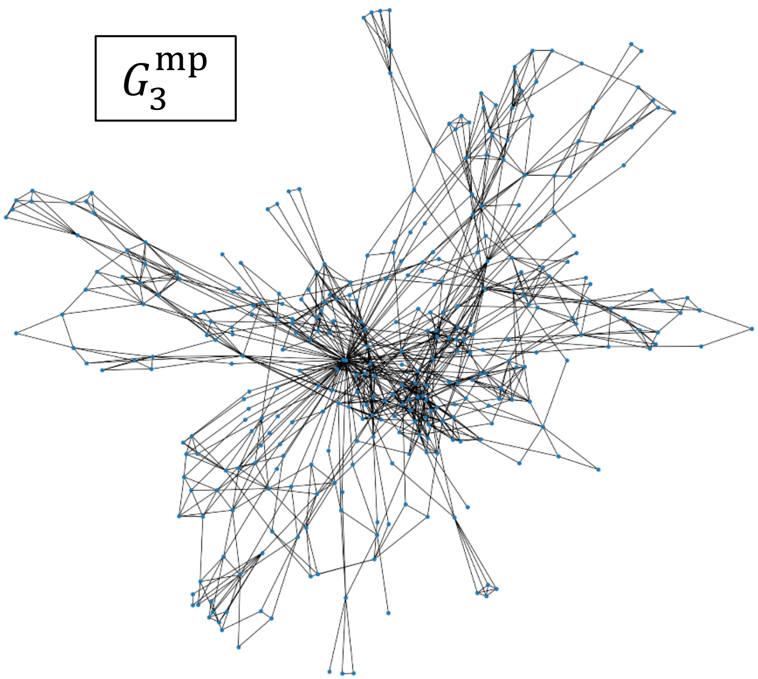
Networks (G_3)

G_3^{md}



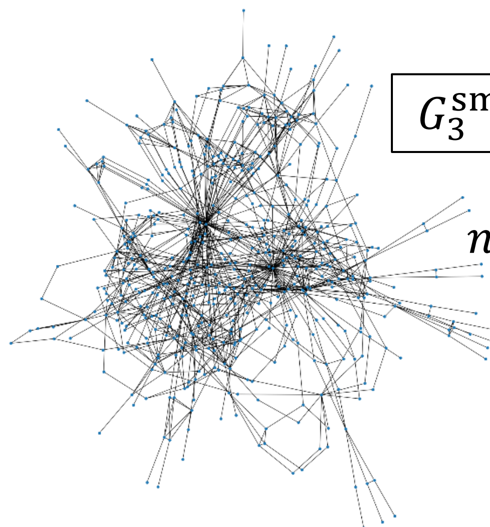
$n = 190, m = 672$

G_3^{mp}



$n = 344, m = 975$

G_3^{sm}



$n = 530, m = 1103$

communication network concepts, network attack, and measures

METHODOLOGY

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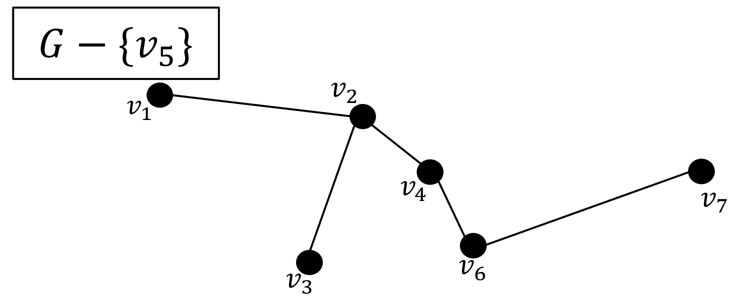
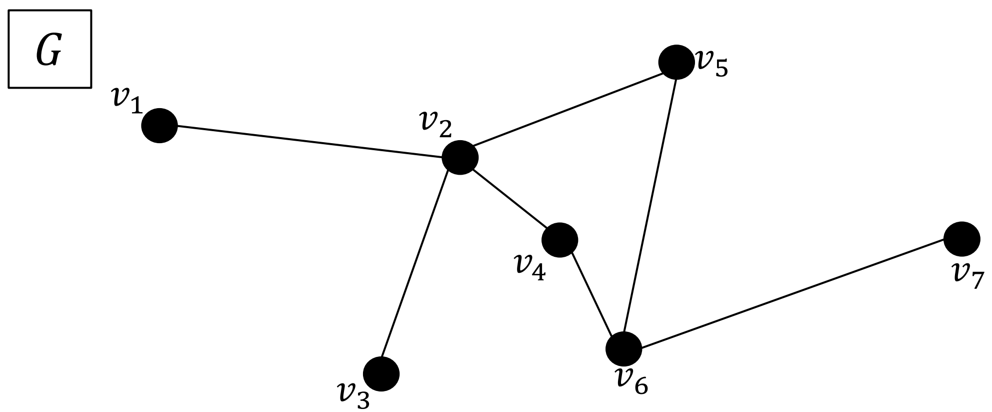
Communication Concepts

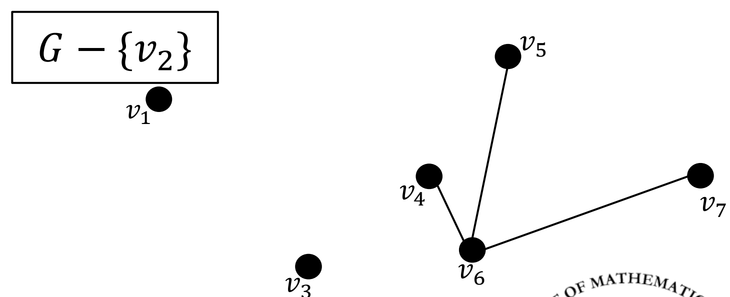
- <u>latency</u> - the "delay" between any two nodes in a network; measurable by distance or edge weight sum; $\ell_G = \frac{\sum_{u,v \in V(G)} d(u,v)}{n^2}$.
- <u>fragmentation</u> - an increase in the number of network components.



$d(v_1, v_7) = 4$



$d(v_1, v_7) = 4$



$d(v_1, v_7) = undef$

Network Attack

Node deletion simulates a network attack. To compare models, we need a standard method to delete nodes: use the betweenness centrality values to delete nodes, going from high-to-low.

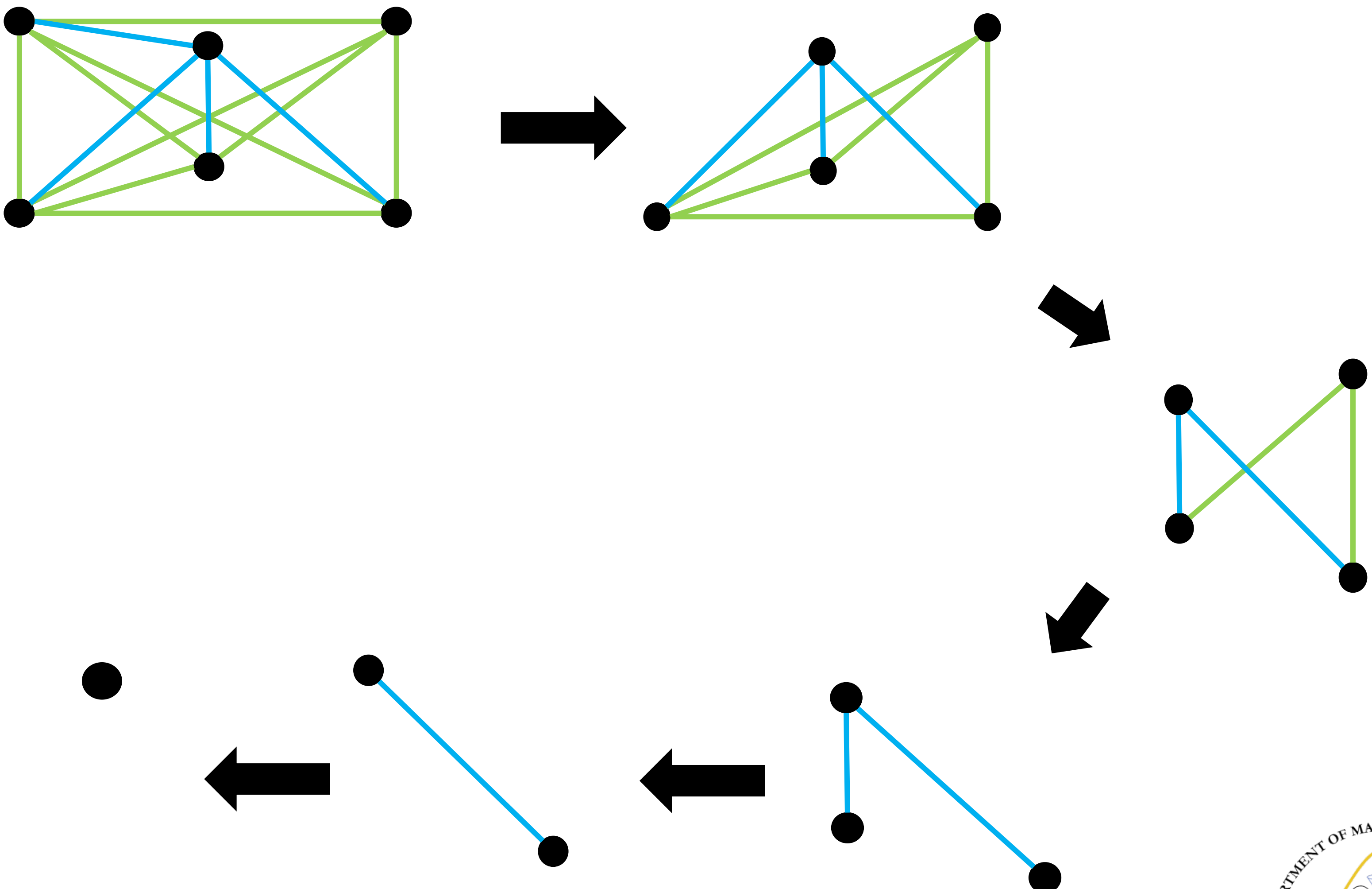
Given $G = (V, E, L)$, let $W = \{w_1, \dots, w_{|V(G)|}\}$ be an ordered bijective mapping of $V(G)$ where the following hold:

- $x_{w_1} = \max(x_v) \ \forall \ v \in V(G)$
- $x_{w_j} \geq x_{w_{j+1}} \ \forall \ w_j \in W$

The centrality list for each model is different. We sequentially delete nodes beginning with w_1.

Note: for all simple-modified models, if we delete a supernode, we delete its neighbors.

Attack



Measures

- $\boldsymbol{c(t_i)} = c(t_{i-1}) + c_{v_i}$, the cumulative cost up to the i^{th} node deletion where $c(t_0) = 0$ and c_{v_i} is the cost of deleting node v_i.

- $\boldsymbol{k_i(G)}$, the number of components following the i^{th} node deletion.

- $\boldsymbol{c_k(t_i)} = \begin{cases} \frac{c(t_i)}{k_i(G)}, \text{if } k_i(G) > k_{i-1}(G) \\ undef, \text{otherwise} \end{cases}$, the cost to achieve further network fragmentation.

- $\boldsymbol{\ell_{sum}} = \sum_{H_i} \ell_{H_i}$, the sum of components' average shortest path.

Measures

- $\widehat{\boldsymbol{m}}_{\boldsymbol{H}} = \frac{|E(H)|}{|E(G)|}$, the relative size of the largest component H of G.

- $\widehat{\boldsymbol{n}}_{\boldsymbol{H}} = \frac{|V(H)|}{|V(G)|}$, the relative order of the largest component H of G.

- $\ell_{\boldsymbol{sum}}^{\boldsymbol{size}} = \sum_{H_i} \widehat{m}_{H_i} \ell_{H_i}$, the sum of components' average shortest path with each weighted by a component's relative size to G.

- $\ell_{\boldsymbol{sum}}^{\boldsymbol{order}} = \sum_{H_i} \hat{n}_{H_i} \ell_{H_i}$, the sum of components' average shortest path with each weighted by a component's relative order to G.

Attacking G_1, G_2, and G_3

RESULTS AND ANALYSIS

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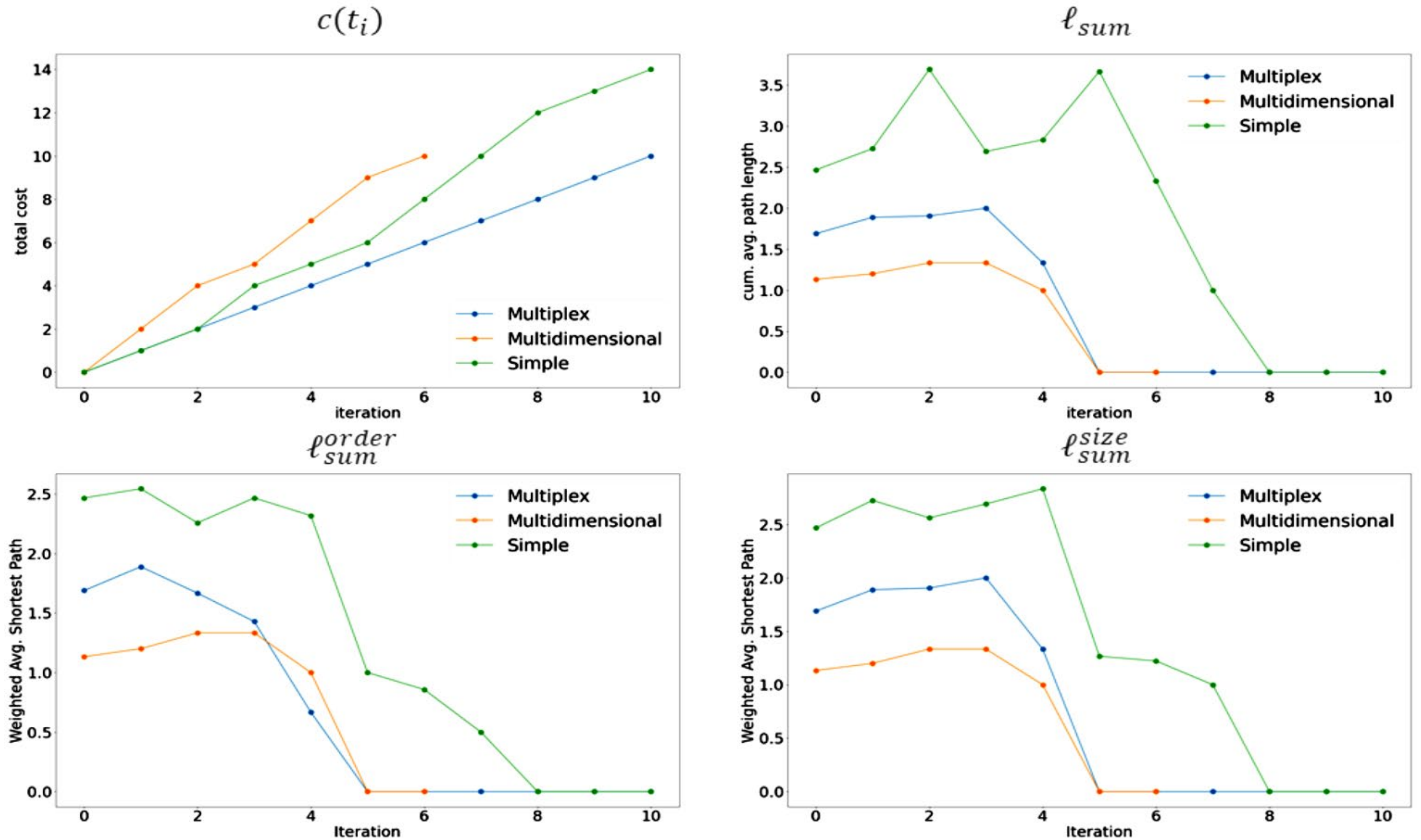
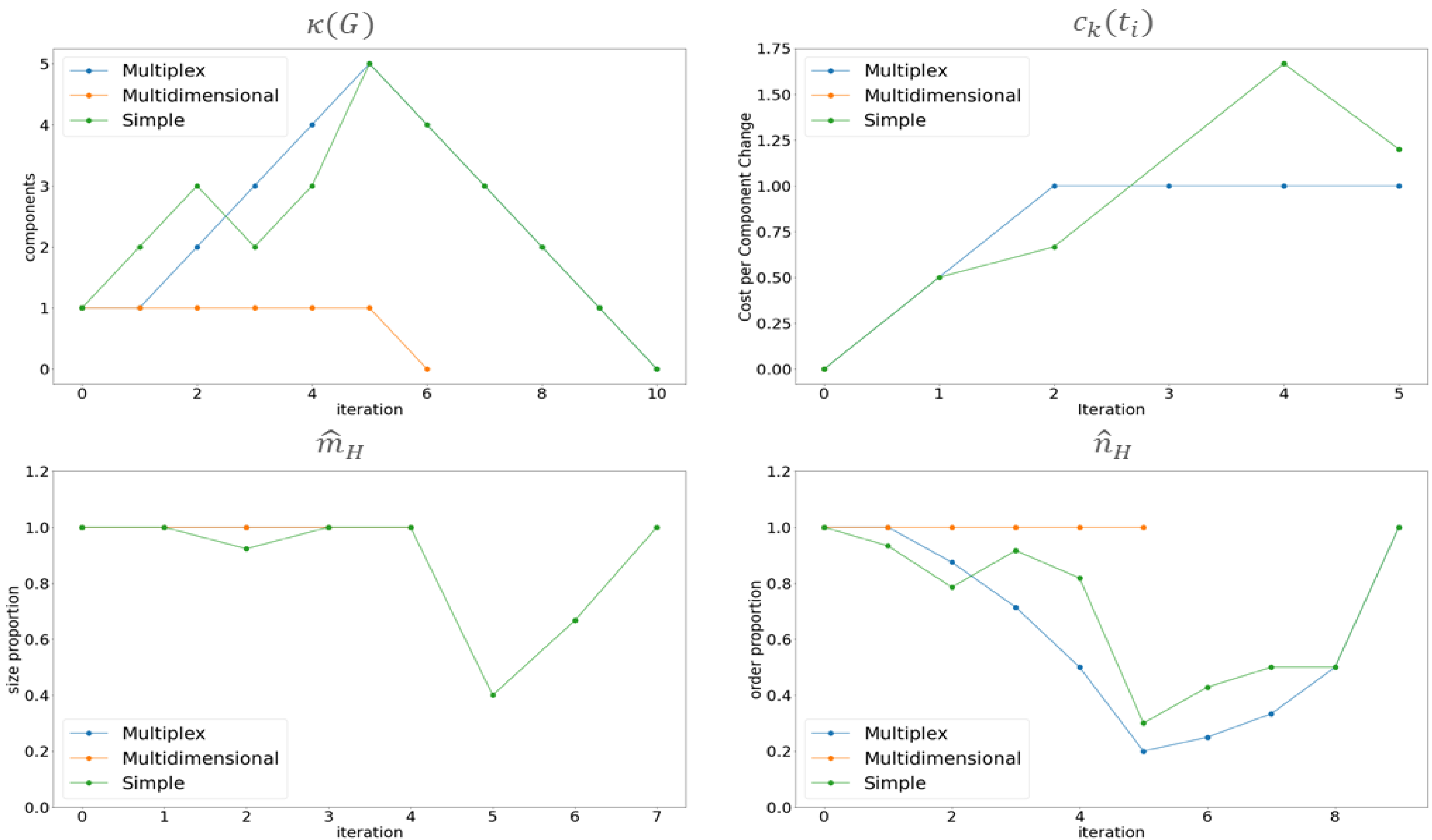
Attacking G_1



- G_1^{mp} has lowest cost overall.
- G_1^{sm} yields highest latency.

Attacking G_1



- Both G_1^{mp} and G_1^{sm} achieve $\max[\kappa_i(G)] = 5$ at the fifth deletion.
- G_1^{sm} achieves $\kappa_5(G_1^{sm})$ at a higher cost than G_1^{mp} does for $\kappa_5(G_1^{mp})$.

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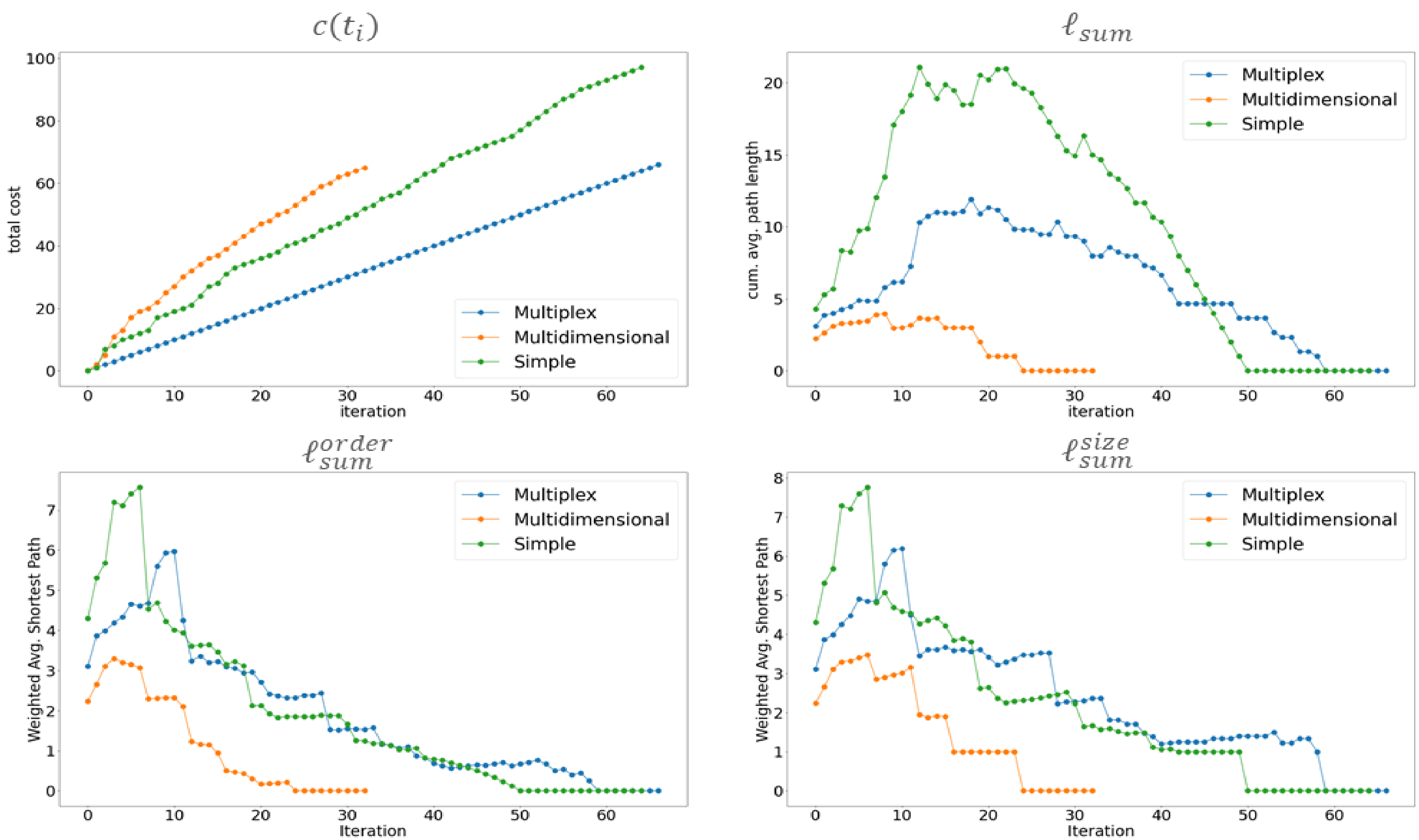
Attacking G_2



- No change in overall cost performance between models.
- Both G_2^{mp} and G_2^{sm} perform similarly for ℓ_{sum}^{order} and ℓ_{sum}^{size}, trading back and forth beginning around the tenth deletion.
- G_2^{sm} dominates ℓ_{sum} until the forty-fifth iteration.

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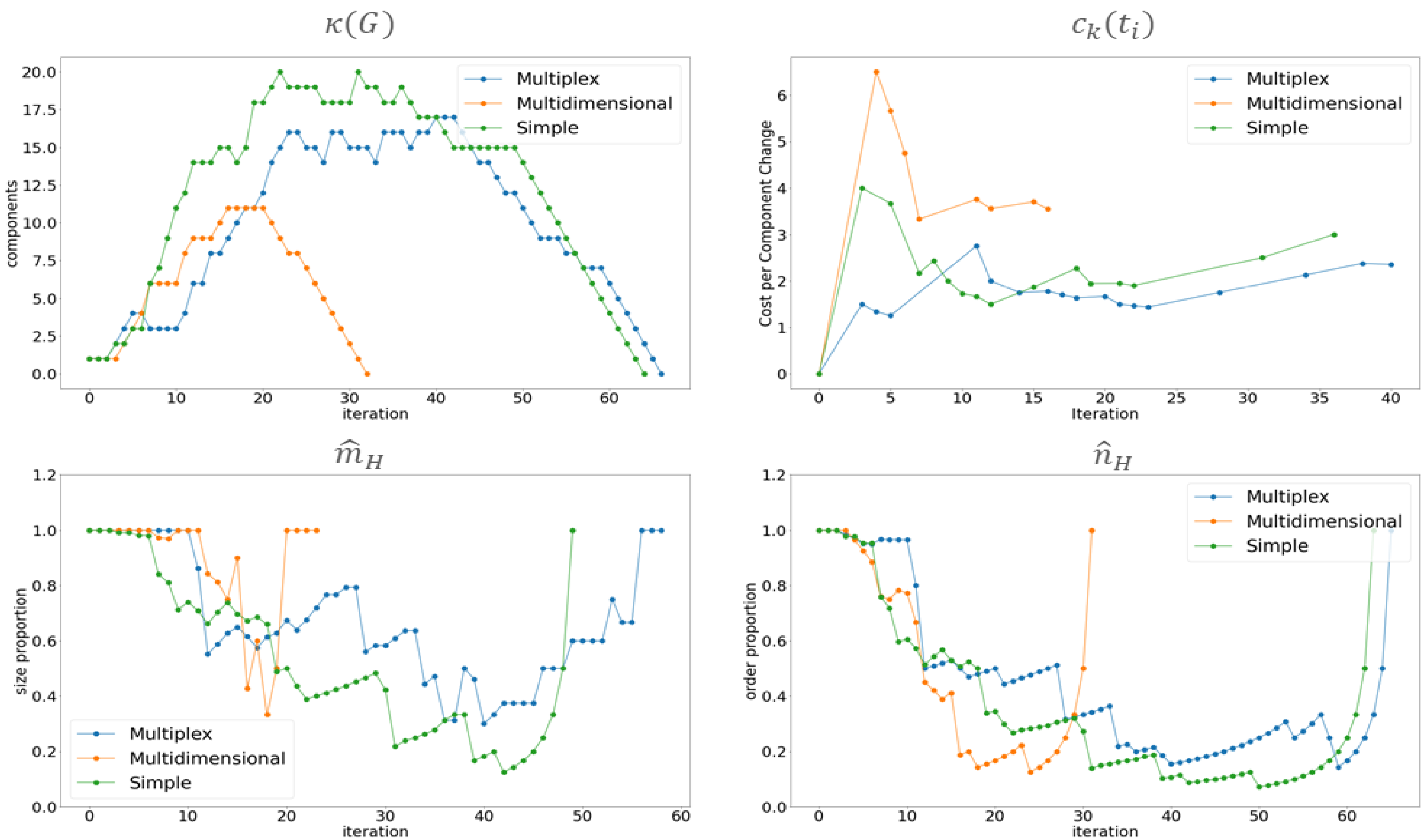
Attacking G_2



- $\max[\kappa(G)] = \kappa_{21}(G_2^{sm}) = \kappa_{31}(G_2^{sm}) = 20$
- $\kappa(G_2^{md})$ generally larger than $\kappa(G_2^{mp})$ until the twentieth deletion.
- Network fragmentation always costs more for G_2^{md}.
- G_2^{md} achieves a comparable minimum $\hat{n}_H$ at 33% fewer node deletions.

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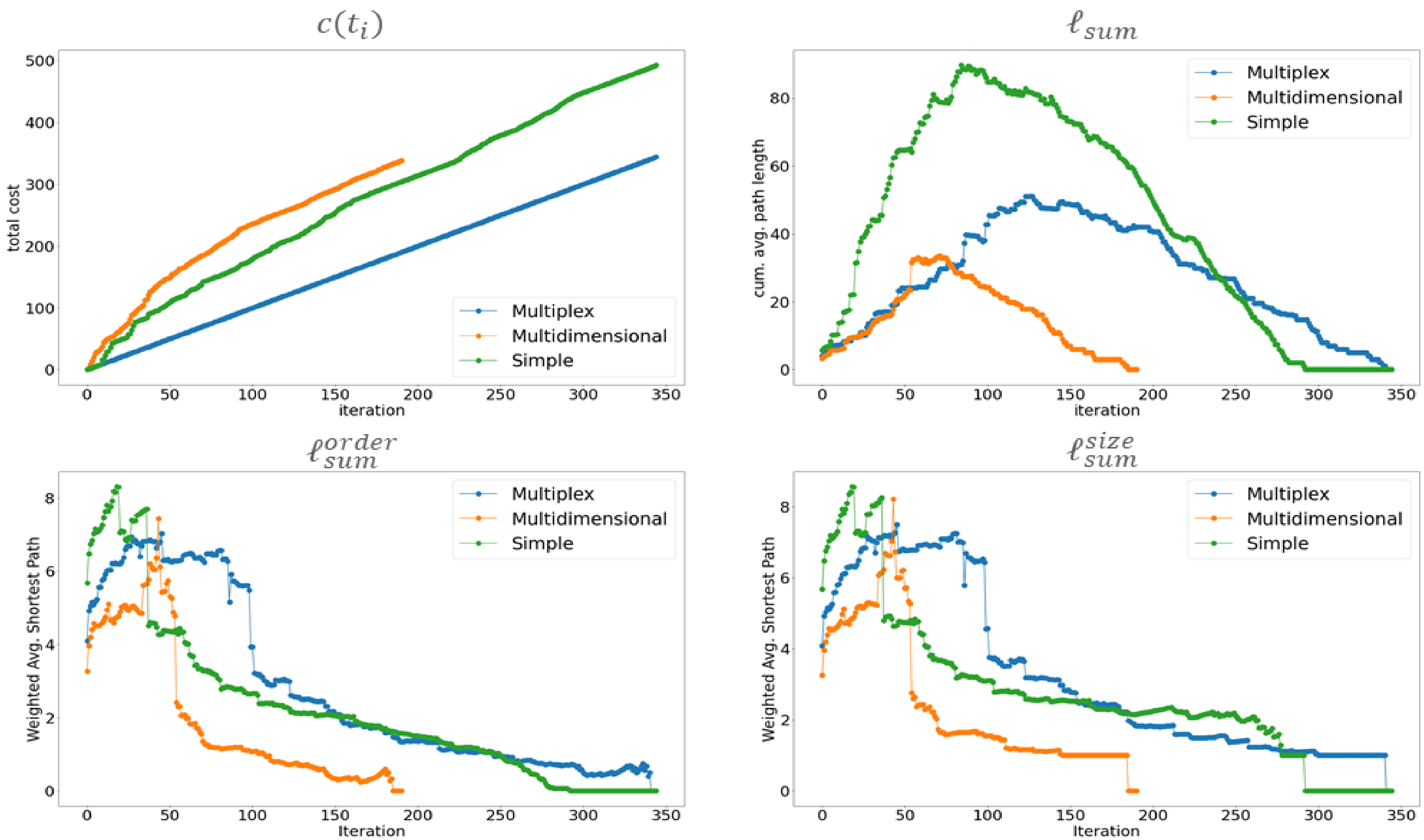
Attacking G_3



- No change in overall cost performance between models.
- Essentially same results for ℓ_{sum} as G_2.
- The ℓ_{sum}^{size} chart tells the same story as ℓ_{sum}^{order}.

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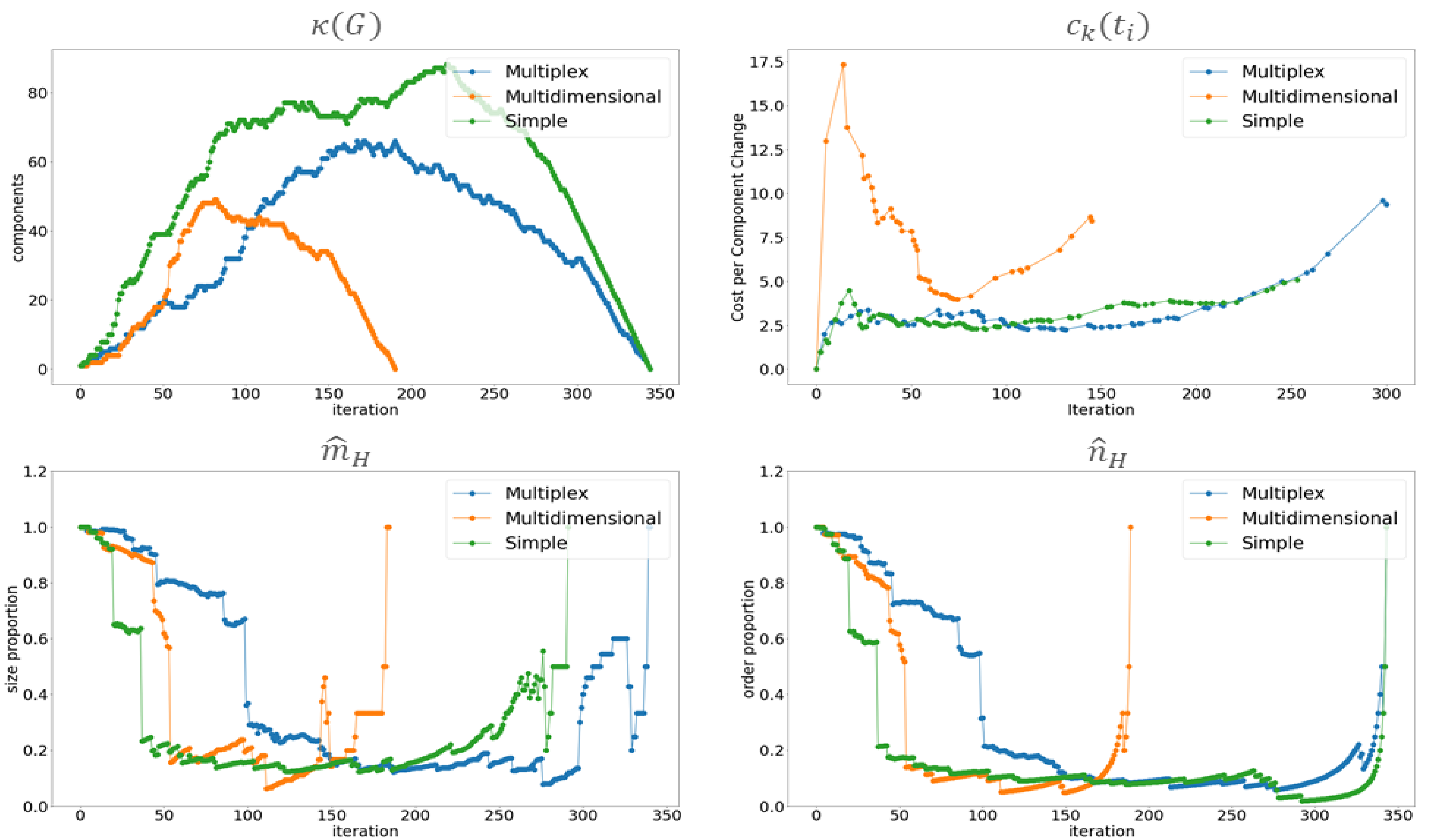
Attacking G_3



- G_3^{mp} outperforms all for fragmentation throughout the deletion process.
- Network fragmentation always costs more for G_3^{md}.
- Both G_3^{sm} and G_3^{md} demonstrate similar performance for $\widehat{m}_H$ and $\widehat{n}_H$ until $\kappa(G_3^{md})$ begins its downward trend.

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summary and future work

CONCLUSION

Summary

	Latency Preference	Fragmentation Preference	Holistic Preference
$\boldsymbol{G_1}$	G_1^{sm}	G_1^{mp}	G_1^{sm}
$\boldsymbol{G_2}$	G_2^{sm}	G_2^{sm}	G_2^{sm}
$\boldsymbol{G_3}$	G_3^{mp}	G_3^{sm}	G_3^{sm}

Future Work

- Compare other modeling methods;
- Adjust/replace measures, e.g., d_W;
- Analyze other types of organizations;
- Alter the simulated environment and equipment distribution;
- Implement other node deletion schemes, including group and adaptive;
- Incorporate probabilistic equipment failure.

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Thank you

END OF PRESENTATION

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