

Know Your Customer: Driving Calculus Course Design through Data

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Abstract – Research suggests that students struggle with calculus on various levels. Student and instructor feedback is a necessary cycle to restructure and improve course curriculum. The aim of this paper is to discuss and illustrate how solicited feedback and administrative data can be coupled together to drive decisions about course design.

I. BACKGROUND

Teaching single variable calculus to students at the undergraduate level presents both challenges and opportunities when it is a required component of the core curriculum. Every cadet at the United States Military Academy (USMA) is required to successfully complete 4.5 credit hours of calculus despite their field of study or mathematical background. To accomplish this requirement, students can test into an advanced math program or be placed into the core mathematics sequence. Single variable calculus is the second course in the three-course core mathematics sequence.

Calculus is often celebrated as being “one of the greatest intellectual achievements of western civilization.”[1] Despite its fame and problem-solving supremacy, research shows that “students struggle to succeed in calculus courses and even successful calculus students often fail to use calculus concepts to solve non-routine problems.”[1] Nested in the middle of the core USMA mathematics sequence, cadets’ success in calculus tends to follow these trends seen in the research.

To combat these trends, the USMA single variable calculus course leadership sought to engage cadets about their attitudes toward mathematics in an effort to better understand various characteristics about the cadets taking the course. Through two separate surveys, the course gathered data consistent with the “Insights and Recommendations from the MAA National Study of College Calculus.”[2] The study conducted by the Mathematical Association of America (MAA) looked at 18 different two and four year colleges and universities to determine who takes calculus, how calculus is

taught, and elements that make calculus programs successful. While this study is useful in determining best practices for designing a calculus course for students who are opting to take it, it does not take into consideration those collegiate programs, such as USMA, where the same calculus course is required for all major fields of study.

The intent of this paper is to illustrate how course leadership can leverage survey and administrative data to design a course to meet the challenges that stem from a diverse set of student backgrounds and attitudes toward mathematics. If a single course is intended to meet the requirements for STEM (Science, Technology, Engineering, and Mathematics) and non-STEM majors, then what is the best way to design the course? We seek to answer this question through data collected from the cadets that take the course and the faculty that instruct it.

II. METHODS

Course leadership provided instructors and students with initial and final surveys at the beginning and end of the spring 2017 semester. The initial survey designed for the cadets and faculty was an adaptation of the MAA National Study of College Calculus surveys that were given to various undergraduate calculus students in the fall term of 2010.[3] The final surveys, however, used several follow-up questions to gauge changes in attitudes as well as new questions to evaluate perceptions of overall student learning. These surveys, when merged with administrative admissions data, provided an overall picture of the cadets enrolled in the course. Similarly, the faculty surveys provided the course leadership with a better understanding of the backgrounds and views of the instructors teaching the material to the cadets.

III. RESULTS

Understanding the Single Variable Calculus Student

The surveys established two characteristics about the typical cadet enrolled in single variable calculus: *overconfident* and *strategic*. Namely, students felt they had more of a mastery of foundational material than they actually did and their efforts to study for course material were aimed at receiving an adequate grade instead of truly understanding the key concepts of calculus.

The first effort in determining the changes we needed to make to the single variable calculus curriculum involved identifying student shortcomings, knowledge gaps, and attitudes that affect their ability to understand the material. Table 1 shows the background of the cadets enrolled during the primary offering of single variable calculus. Cadets were asked about the calculus related courses they had taken prior to entering USMA from a list of possible choices that included AP calculus, other college calculus courses and pre-calculus. As the table shows, 93.7% had taken some level of pre-calculus while 73% claimed to have taken an actual calculus course. Further questions from initial surveys given to cadets prior to taking single variable calculus indicated that cadets generally maintained a lesser view of their own abilities in mathematics versus how they felt others viewed their abilities. Prior to starting the course most students felt prepared to take on the challenges of calculus. In fact, 90% of students felt that they had the abilities to succeed in single variable calculus and in a separate question, 80% of students responded that they felt prepared to take Calculus I.

Math Courses Cadets Took Prior to Attending West Point		
Course	Cadets	Percent
Pre-Calculus	770	93.7%
Calculus (Non AP)	189	23.0%
AP Calculus AB	344	41.8%
AP Calculus BC	68	8.3%
International Baccalaureate (IB) Calculus	20	2.4%
Online calculus course	8	1.0%
Calculus course taught at my high school for which I also received college credit	36	4.4%
Calculus course taught at a two-year college for which I also received high school credit	13	1.6%
Calculus course while I was enrolled in college	65	7.9%
Calculus course taught at the USMA Prep School	46	5.6%

Table 1: Cadets self-reported mathematics background prior to entering West Point.

Throughout the initial surveys, cadets were generally optimistic about their mathematical ability. They felt that they could be successful in a single variable calculus course; in fact, 80% of cadets responded that solving a problem involving mathematical reasoning was a satisfying experience while 65% of cadets indicated that they enjoy

mathematics and 60% stated that they would take single variable calculus even if it were not required. Despite the optimism, final grades showed a disparity between expectations and reality as seen in Table 2, primarily among the cadets who thought of themselves as A and B calculus students.

	Actual Grade					
	A	B	C	D	F	
Expected Grade	A	114	212	93	7	6
	B	5	94	208	26	19
	C	2	1	24	6	16
	D	0	0	0	0	1

Table 2: Number of students who expected grade X that received grade Y.

The survey presented to cadets at the end of the course was designed to gain insight into how the cadets viewed certain aspects of the course as well as how much the cadets learned throughout the semester. The survey indicated that 84% of cadets view the primary role of a math instructor as helping students reason through problems on their own. Most cadets conveyed that they understood the course material through classroom instruction, working through board problems (a pedagogical approach where instructors have a class work through problems on individual chalk/white boards), and collaboration with their classmates. When asked whether the textbook was useful in understanding the material, an overwhelming 76% said it was not useful and in a separate question, 88% of cadets said that they preferred online video instruction to the textbook.

Several questions on the end of course survey revolved around the way in which the cadets view their own learning and how they think. In response to these questions, 64% of cadets admit to be strategic learners (learning in order to perform well on exams, but failing to retain material afterward) while 75% of the cadets consider themselves deep thinkers. These two responses seem to present contradictory undertones; deep thinking is more likely to be related to deep learning rather than strategic learning. If over half of calculus students admit to focusing their efforts to learn for the sake of performing well on exams while failing to retain the material, then the fact that 75% of the students claim to be deep thinkers is quite suspicious and begs some questions about the students' efforts to learn.

Understanding the Instructor

The instructors teaching single variable calculus at USMA are comprised of a conglomeration of rotating military faculty, permanent military faculty, and civilian instructors and professors. The single variable calculus course typically leverages 20 to 25 instructors for the primary semester that it is offered. Of the instructors who taught the course during the primary semester of the 2017 academic year, 22% said that they would not request to teach it again. This high of a percentage could be a result of the many requirements a 4.5 credit course imposes on instructors; however, this percentage requires more attention as outlined below. The complete breakdown of instructor responses for teaching calculus in the future is listed in Table 3.

Requested	Would Request Again	
	Yes	No
	Yes	No
	63%	11%
	16%	11%

Table 3: Rounded percentages of responses for instructors requesting to teach calculus.

To understand the data presented in Table 3, the instructors were asked a series of questions to gauge their satisfaction of the course and their views on teaching single variable calculus through free text responses. The general feeling among instructors was that they enjoyed the freedom they had to teach the course as they saw best and they felt that the course leadership provided them with the necessary resources they needed to succeed in the classroom. Each section took the same exams and completed the same projects; however, instructors had flexibility to use quizzes, homework assignments, and in-class activities as they saw best. One instructor commented that “this was a great highlight of [the] course” while another stated that he was “empowered by leadership to adjust [his] classroom to [his] teaching style.” A separate instructor, however, noted that his satisfaction of the course was not as high as he thought it would be, noting several sentiments felt by numerous faculty: “I love math and the calculus, but the fast tempo combined with the wide range of student abilities made teaching the course less enjoyable than it might have otherwise been.”

To learn single variable calculus students must work through problems to understand the foundational concepts. The course leadership pushed the importance of performing “quality repetitions.” Athletes learn different skills through repetitions and drills that are designed to build fundamentals;

they further their skills by building upon these foundations through more repetitions and drills. This mindset was taken into the single variable classroom. In a typical single variable calculus course at West Point, cadets solve problems at individual chalk boards while the instructor assists cadets who are struggling as well as answer more in depth questions to students who are at a more advanced level. When instructors were asked about how well their students performed quality repetitions, typical responses were “not well”, “bad”, and “poorly.” One instructor wrote, “I feel that my students almost universally refused to do quality [repetitions] on a constant basis. Before major graded events, students tried to squeeze in quality [repetitions] to force understanding. When this method failed, they blamed the process instead of the lack of quality.”

Despite the frustration instructors had with cadets not being able to perform at a level they thought they should perform, most instructors maintained a positive view of the course and the way in which it was run from an administrative level. General feedback revolved around the pace and the amount of rigor in the course. One instructor cited a tendency in course leadership to increase the difficulty of problems year to year, essentially forgetting that each year a new batch of students arrive with the same level of understanding.

The overall feedback from the instructors supported the narrative that students desire to pursue strategic versus deep learning while possessing a wide range of mathematical foundational abilities cloaked in optimism. The rigors of West Point extracurricular requirements undoubtedly play a role in student fatigue and performance that may not otherwise be a factor at other universities; such requirements may certainly redirect an otherwise deep thinking cadet to take a strategic approach to academics.

Redesigning the Course

Through the use of the surveys and cadets’ administrative data, the course leadership made a series of changes aimed at overcoming cadets’ tendency to strategically learn while attempting to reduce the variance in mathematical foundational abilities. The first change was to standardize the meaning of “quality repetition.” The course leadership defined a quality repetition of a problem as the incorporation of five descriptive steps for solving a calculus problem: problem statement, assumptions, formulation, methodology, and results. This standardization of a “quality repetition” aimed to combat against strategic learning. Requiring students to implement a quality repetition forces them to look at the problem deeper and gain a more

comprehensive understanding of why they are taking certain steps to solve a problem.

The second change was to reduce the size of individual problem sets in order to increase the quantity of problem sets. The primary effort was to implement problem sets that provided a more comprehensible real-world context and to do them more frequently with less complicated problems. The problem sets took a military history context using a Combat Outpost in Northeast Afghanistan. The nature of the problems allowed cadets to make connections to the mathematics using their own experiences from their cadet basic training. The course leadership felt that by having these connections and some level of interest in military scenarios, they could gain a better understanding of the applicability of single variable calculus concepts. In conducting more problem sets, cadets could also improve their ability to communicate the results while furthering their level of understanding of the material.

Additional course changes included a week of lessons devoted to functions as well as a self-assessment of algebra, trigonometry, and geometry skills. The restructuring of the first week was designed to target the overconfidence that cadets had going into the course while motivating the need to refresh and study pre-requisite material. Cadets could see what skills they were weak in at the beginning of the semester so that they could adjust their studying accordingly. The cadets needed to understand that calculus requires strong fundamentals of lower level mathematics and that any weak understanding will present challenges later in the course. The course syllabus changed to also incorporate a numeric, graphic, and algebraic approach to present calculus concepts. This adjustment was again aimed at the overconfidence that some students had coming into the course. This approach hits multiple angles of the same calculus topics, which targets both the 73% of cadets who have taken calculus before and those cadets who have not by deepening understanding while scaffolding the concepts.

IV. CONCLUSIONS

The cadets at West Point come from a variety of backgrounds with high aspirations and goals of being successful. As faculty, we expect them to be high achievers and we expect them to have more foundational abilities than they actually do. As such, course leaders must continue to make efforts to collect data from students and instructors to understand how they can make adjustments to support their clientele; the student. Every change made to the course should be to support lessons learned from a careful analysis of feedback and data.

The calculus course at West Point underwent significant changes in order to specifically target two observations of the student. The data collected from surveys indicated that the 2017 calculus student at West Point was overconfident in their mathematical abilities as well as motivated to learn only for exams while failing to retain critical concepts. In making these inferences about the student, the course leadership made changes to problem sets, day-to-day activities and the overall curriculum.

Future efforts of course design should always include an analysis of the student. Ultimately, the student is the customer and to build the student into a lifelong learner and contributor to our organizations and societies, we must understand who they are, where they come from, and what they truly need from a course to be successful. With regard to a single calculus course that meets the needs of all academic majors, this task is challenging. While our current efforts are not perfect, they will continue to be refined through a process similar to that outlined in this paper.

REFERENCES

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