EDITOR'S NOTES

As I complete my tenure as the Editor-in-Chief (and acting Managing Editor) of Mathematic Militaris, I must reflect on a theme that appears to have been consistent for the last twelve years. I noticed recently that an outside reader would get the impression that at the Federal Service Academies, we never make a mistake! All of the great things we have ever done in terms of lively applications, outreach, technology, computer algebra systems, evaluation and assessment, modeling, and numerous other topics have filled these volumes with a consistent theme of "success."

My personal experience in education has been one where the successes are easily outnumbered by the not-so-successes, and I thought it might be fruitful to dedicate a volume to such incidents. Therefore, in this volume we talk about some of the mistakes made in our journey to infinity and beyond, in the hope that we might learn from them.

Not coincidentally, we have chosen to dedicate this issue to this journal's founding Editor-in Chief, BG (Ret) David C. Arney, and our first article discusses his numerous contributions to the mathematics program at USMA. We have many articles that include "lessons learned" from a diverse collection of authors, as well as a very nice article by Father Gabe Costa (USMA) in appreciation of an old teacher whose dedication undoubtedly prevented many future mistakes by his vast collection of students.

I would like to thank the editorial board for their support at the other academies—you have certainly helped keep this endeavor on track, and I wish you all the best as I head to the Pentagon.

Phil Beaver

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SUBSCRIPTIONS TO *MATHEMATICA MILITARIS*:

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Brigadier General (Retired) David C. (Chris) Arney, Mathematician

Lieutenant Colonel Philip Beaver
United States Military Academy

BG Arney retired from active military service on 31 March 2001 after nearly 30 years of distinguished service. During the past 21 years he made enormous contributions to the United States Military Academy through his pursuit of excellence as a leader, soldier, and scholar. He directed significant innovations in strategic planning, faculty development, pedagogy, and curriculum development. He was responsible for a multi-million dollar academic development project involving over 20 academic institutions and coordinated numerous educational activities and programs.

BG Arney directed technical research for numerous Army agencies, including building a major research center for the Academy in coordination with the Army Research Laboratories. BG Arney taught over 65 sections of 23 different courses; created several new academic courses; advised 18 cadet senior research projects; published over 75 technical articles and over 75 book and software reviews; authored or edited 18 books; taught over 30 faculty development workshops and mini-courses; edited over 120 book reviews, over 50 educational projects and modules, and over 60 problem solutions; reviewed and refereed over 100 manuscripts; and given over 150 technical presentations. This includes the time he served as the Editor-in-Chief of *Mathematica Militaris* from its founding in June 1989 through the Fall of 1992.

BG Arney was appointed Program Director for the new core mathematics course in discrete dynamical systems. He implemented it as a core course in 1989. In 1992, BG Arney was selected as a Professor, USMA and began service as the Deputy Head for the Department of Mathematical Sciences. After judging papers for the Mathematics Competition in Modeling for a number of years, he was appointed associate director of that international student contest in 1990. In 1999, he became the founding Director of the Interdisciplinary Contest in Modeling. In 1994, BG Arney was appointed the Acting Department Head, Department of Mathematical Sciences, and in 1995 was appointed Department Head. Throughout this period he enhanced the reputation of the Academy and Army through his research, consulting, and collaboration with army laboratories and civilian universities. In 2000, he was awarded the Mathematical Association of America Distinguished Teaching Award during a semester when he wasn't even teaching!

As a key member of the Academy's leader team, BG Arney directed several large grants from the National Science Foundation and other academic support foundations to improve the academic culture in science and engineering programs. He led his department of 67 faculty members in the teaching of over 30 courses and 5000 cadets annually. Since 1994, he has built a nationally recognized department, which includes three centers of excellence, a nationally supported faculty development model, a national curricular model, and leadership of a consortium of 20 schools devoted to improving interdisciplinary education.

It is rumored that one of the last commands BG Arney issued on active duty was "Beaver, I dare you to publish a *Mathematica Militaris* volume on mistakes, and dedicate it to me." I am proud to thus be a small part of his great legacy.

Lessons Learned by a Course Director

Major Gerald C. Kobylski
United States Military Academy

During my assignment at USMA, I have had the wonderful opportunity to be a course director for three semesters in two different courses. For two semesters I was the course director for MA205, Multivariable Calculus, the third math course in the sequence of four required courses for every cadet at USMA. This course usually has an enrollment of around 850 cadets in the fall and around 80 cadets in the spring. The other course I directed was one of our Department's electives, Nonlinear Programming, MA381. The enrollment for this class was 50 cadets. What follows are some of the lessons I learned from these experiences; the first five lessons are from MA205, and the final one is from MA381. I have certainly learned many more lessons than those outlined here and would be happy to share those with anyone who might be interested.

Homework Sets

We incorporated a graded homework set into MA205 in lieu of an exam. The hope was to reduce the leadership time in preparing an exam, to reduce the grading time for instructors, and to give the cadets the opportunity to work on more application problems. Four groups of instructors developed their own sets. Leadership preparation time for the homework sets was a little bit less than for an exam. Afterwards, all but one instructor felt that the grading was immensely reduced. Cadets had one day to complete the set. The reason for this was twofold: this was a substitute for an exam and we tried to design this exercise to take two hours.

Not surprisingly, the cadets were very positive about the homework set. They felt they were able to learn the material much better because they were able to focus their study time and they were able to ask the right questions of their peers. In most cases this was very good; as would be expected, however, there were cadets who did very little work. Indeed, there was a lot of group work on the homework set. Thus, a homework set is not as good an assessment tool as an exam. Several instructors felt that many cadets got the maximum grade because they compared enough answers, and that cadets who worked on it themselves were at a great disadvantage. One of the instructors commented that the homework set was good if one of the purposes was to raise grades and morale. He added that it forced cadets to work harder than usual, but did not reflect their immediate knowledge as an exam would.

Finally, some cadets' projects took a little longer than the planned two hours. The reason for this was because Mathcad (the Math software we use at USMA) was needed to solve some of the problems. As a result of several instructors commented that we need to teach Mathcad better, or not require its use during timed events.

Lessons Learned. The benefits from group work in learning are enormous and far outweigh the disadvantage of some cadets doing very little work. Instructors agreed though that the cadets should have had at least two days. A purpose of any homework set should not be assessment. Therefore, the rationale for just allowing one day is not valid. Instructors need to understand that a homework set is a way to maximizing learning through group work, and that assessment is secondary.

Instructors enjoyed the autonomy in coming up with their own problems for the homework set. If different homework sets are distributed to the course, extreme care must be given to ensure that there is uniformity with respect to level of difficulty and time. Finally, in order to motivate cadets more in learning the material that the homework set covers, the next exam could include the concepts from the homework set; the homework set need not be in lieu of an exam as it was in our case.

Modeling Lessons

MA205, Multivariable Calculus, has historically been a "topics" course in that it does not really "dig in" to some concepts and show neat application problems. I deleted several major topics such as cylindrical and spherical coordinates and triple integration in order to make room for four modeling and discovery lessons. I timed these so that the cadets could digest material a little more by using their newly learned skills to model realistic problems and then solve these problems. The course guide included several problems for the instructors to use for each of the four lessons. Instructors had the flexibility to give quizzes during these lessons and many took advantage of this opportunity. These lessons went a long way in adding more depth to the course. However, I do not think as many instructors made it to the application problems as I would have liked.

Lessons Learned: These four modeling and discovery lessons slowed the pace of the course a bit. It is difficult to delete topics from a core mathematics course because so many future courses depend on basic mathematics skills. Therefore, when we do make room in the syllabus for the purpose of modeling, the focus of these lessons should be solely on modeling and discovery (no quizzes). These lessons should not be a time for instructors to catch up. Instructors also need access to a forum where they can share interesting articles or Web sites about applications. We began something like this with the hopes that it would be built upon over the next few semesters.

Oral Presentations

I introduced an oral presentation into the course for the projects to motivate the teams to work on the entire project together. I felt that although we emphasized communication skills in our course, besides briefing board problems, cadets do not get enough opportunities to give some form of an oral presentation in the course or at the Academy. Another reason for the oral presentation was saving the cadets time; they spend a lot of time putting together the formal report which we currently require, sometimes more than on the mathematics involved in the project. Developing slides was much more time-efficient for the cadets. A final benefit I hoped would result from this exercise was that it would reduce the time instructors spend grading projects.

Like any exercise, some groups put more time into their briefing than others. Many groups, however, did a great job with the presentation. Positive cadet feedback was overwhelming. First and foremost, they commented on how the possibility that they might have to answer a question on the project motivated them to be involved in the project. This generated more student cooperation and teamwork than on written projects; students could no longer divide up the write-up of the report. Second, the cadets knew the importance of being able to give a good brief in the Army and that this was great practice for them. Finally, most said that it was not as time-

consuming as putting together the technical report.

The first time I tried this technique, instructors felt a little uncomfortable when grading the oral presentation. In order to better prepare instructors for this difficult grading, I scheduled some time where one instructor gave a sample presentation to all of the instructors and we all were able to look at the grade sheet and discuss this presentation. The instructor did an outstanding job with the presentation. As a result, there was little to discuss how he could have done it better. One of the main points that came out of this assessment exercise was that the instructors should give the cadets some theme to focus their briefing on. Cadets should not try to brief their entire project but just meet the intent of the instructor.

Lessons Learned: In order to better prepare instructors and to try to standardize the grading as much as possible, the sample presentation given to the instructors should have a lot of intentional flaws; it should be "C" work. This can lead to a better discussion of what the instructors might see and how they should grade accordingly.

Mathcad Skills

The primary area where cadets used technology (Mathcad) in the course was in the two course projects. One of the goals for these projects was to show the benefits of using technology in solving application problems. This was accomplished by giving the cadets more realistic problems that were difficult and time consuming to solve by hand.

Many students will not learn a math software program until the time when they absolutely must; unfortunately, for many this is the night before the project is due. The challenge instructors encountered was to prevent procrastination in learning the Mathcad skills. Additionally, instructors had to decide how to best teach these skills given the time constraints in class.

Just as with other math software programs, Mathcad performs many of the operations that we studied in the course. Some of these skills, however, are too difficult to learn for the value of the understanding they provide the cadets. I developed a set of 11 Mathcad skills that each cadet should know in the course and pinpointed for the cadets and instructors each lesson these skills applied. Instructors were highly encouraged to cover these in class and then were given a certain number of points to assess the cadets on these skills. Doing this greatly focused instructors' teaching and student learning throughout the semester.

The results were that cadets gained much confidence in their ability to solve problems using their computer and were a step ahead when project time came around. The cadets discovered that they could solve real-world problems, some which would be very hard to solve by hand, in minutes. Additionally, they could in seconds address the validity of any assumptions they made and answer other "what if" sensitivity questions. As a result of their competence in Mathcad, the time they spent on the two projects was more in line with our time estimates; in previous semesters we had grossly underestimated the amount of cadets hours spent on these projects.

Lessons Learned: After teaching these 11 skills in two semesters, I recommended that we further narrow the skills down to five. These skills would still give the cadets a more in-depth understanding of the Calculus concepts and show the benefits of using technology in solving application problems. Additionally, these skills do not require much teaching time.

Performing these skills on the computer gives the student much more capability to solve realistic problems involving complicated derivatives and integrals and complex equations that either are difficult or time consuming to solve by hand.

Computer Exercises
I asked that instructors use 50 of their 500 instructor points during the first sectioning on Mathcad assessment. Having short homework exercises that focused on these skills helped the cadets become more confident with their Mathcad abilities. The homework also motivated them to learn the skills gradually during the course, rather than learning all of them the night before the project was due. When we resectioned at Lesson 37, there was no Mathcad point requirement for the instructors. As a result, I do not believe Mathcad was discussed as much as I would have liked by the instructors.

Lessons Learned: If cadets are going to learn Mathcad as the course progresses, then instructors should periodically assess their skills. Assessing the five skills I have outlined above would be easy on drill problems. A perfect example would be when setting up double integrals for center of mass. The cadets could solve these using Mathcad.

Emphasis on Conceptual Understanding
In the nonlinear programming course I taught, there was a lot of emphasis on the conceptual understanding of the algorithms and techniques studied; more than half of each exam and quiz focused on this type of assessment. As expected, some students did not like this and in some cases, were not prepared for this.

Lessons Learned: Most of my lectures and discussions focused heavily on the conceptual understanding with little emphasis on the procedural nature of the mathematics; analysis of Maple graphics was a common daily practice. I left the practice of the procedures for them to do for homework (I always showed at least one example in class). I should have structured the homework sets so that there were more "explain" questions rather than just procedural problems. This might have better prepared them for the level of understanding I wanted them to attain.

Misteaks Can Teach You Things!

Professor Brian Winkel
United States Military Academy

It is said in some circles (other geometric configurations as well) that we learn most when we make mistakes. Whether or not "most" is appropriate is not the issue. It is the fact that we do learn from mistakes, and not just with regard to attempts not to make them again. I relate several mistake situations in my career, some significant, some frivolous.

Lessons from managing academic journals
I edit two journals, *PRIMUS* and *Cryptologia*. I have been doing the latter for over 25 years. When an issue comes back from the printer I usually open a sample copy, scan it with some pride, feel the "stuff" of an academic journal, gaze at the covers, check the binder, touch the edges, etc. In the past I would also open to a page at random and read, just read. Many times I would find typos, mistakes in the setting of the text or an equation. I vowed to do something about it and I did. I never do the reading part anymore! I just

open the page at random and then close it!!! From those mistakes made –the typos and also reading to find them, I vowed to work harder to eliminate error, but I know such things will happen and as hard as I try, the journal will not be free of errors. Indeed, years ago, after one of my bouts of "depression" at finding such typos from these random reads, my fellow Cryptologia editor, Greg Mellen, sent me a full back page ad for automobile sales from a Minneapolis newspaper in which the top headline read in full 48 point font, "PUBLIC AUCTION," only the "L" was missing. So I could do worse, but I will try better to minimize the typos and I will not make the mistake of random reads after the fact.

I have learned from my time here at West Point that it is better to be direct, to be up front, and to be prompt, than to wallow in indecision, to delay, to put off. I have learned this from my association with military folk here, from my own aging and maturing(?) process, and from a mistake I made a few years back. In Cryptologia we published a book review, a very critical review, of a vanity press book in which a "secure" cipher system was advocated. Incidentally, after the initial publication of the book the book's author came out with a 50 page errata list!!!! The author was incensed at the review and demanded an apology. Reviews are reviews. They contain opinions, but have facts that can be substantiated – these were! We stood by the review, but offered the author a chance for rebuttal. He ranted about lost sales and issued veiled threats, but never came forth with a rebuttal. The author of the book wrote to the Dean at West Point saying that "There was a dishonorable faculty there in the midst of an honorable institution." He went on to describe to our Dean his concerns. The Dean sent the message to my department head (chain of command, you know) who shared it with me and asked me to look into this and clear it up. While I was working up yet another letter of appeasement to the author, another missile was launched at our Dean by the book's author, suggesting that he would be contacting his Senator about this dishonorable individual at the Academy. The Dean now suggested to my department head that I "get rid of this crack pot." So my position was understood by my chain of command. However, I still had the author to deal with. Each time I would send a new letter of appeasement I got back a nastier letter with more accusations, all of which I shared with my fellow editors. Finally, I got a copy of a hand written letter from one of my editors to this particular author. The editor's letter began, "I just read your latest whine . . ." and went on to blast away at the author for unprofessional conduct. I had made a mistake in thinking "the customer is always right" and sending letter after letter of appeasement. The customer is not always right and sometimes needs to be told so. Such telling should be direct, up front, and prompt. The mistake of pandering to this author was mine, but it will not be made again.

Issues involved in parents of students relationships

During the spring of my first year of teaching in a liberal arts setting I received a letter from concerned parents. They "demanded" a meeting with me to talk about their son's performance. I wrote back and suggested the following; "Since you are arriving over a lovely spring weekend I suggest we meet at the third base seats at the baseball field at 2:00 PM Saturday, for the college has a good team. We could enjoy a good game and have a good conversation. I shall have a gold corduroy jacket." They never showed up!! On

Monday when I told my chairperson (he was not at the game!) about this he laughed (at me!) and we had a frank discussion about decorum and the proper way to respond to such parents. I did meet with them several weeks later, after calling and apologizing, and we did our business in the office, across the desk, in the manner in which they would expect a PROFESSOR to conduct him/herself. I learned to put myself in their shoes, to take their concerns seriously, to say that while customers may not always be right, they need a respectful hearing in a zone in which they are comfortable.

Issues involving teaching

While at Rose-Hulman Institute of Technology, I was part of a team that developed a revolutionary way of teaching all the first-year science, engineering, and mathematics coursework – put 60 students together with 5 faculty (mathematics, computer science, physics, chemistry, and engineering) into one giant team-taught, 12 credit course and integrate the science, engineering, and mathematics whenever and wherever possible. We had been planning the course for over two years, including a delay because of National Science Foundation hold-ups on awards. So when we finally "unleashed" on our first class we literally almost killed them. We came on like the Furies, we assigned design projects from the start on top of homeworks with no class time devoted to discussions of the projects, we did all of our correspondence with them electronically, posting all the material on an internal site (this was at the infancy of email and networks), and we assigned them tough, challenging, and integrating problem activities. After three days of class we CALLED-OFF class for the next two days and locked the computer labs. We had a "town meeting" with the students and let them vent their frustrations. They said things like, "Give me a piece of paper with the homework on it." "We know complex problems can teach us more than simple drill problems, but you should not replace 10 drill problems with 10 complex problems." We had made a mistake, BIG TIME!!! 5 smart (well???) PhD's whose ideas had convinced foundation after foundation to give us money to try out our theories, did not have an ounce of common sense between us when it came to visualizing the onslaught we were planning to perpetrate upon our students.

We were guilty of an overzealous attack on the students' time, on their intellectual abilities to absorb and function, and on their confidence in us to teach them in this new setting. We had planned too much, assigned too much, demanded too much, pushed too much, etc'ed too much! We needed to back off and they had told us with their actions as well as with their ebbing energy levels as they honestly tried to meet our absurd expectations. We listened, we changed, we did back off, we became more reasonable in view of our coming to realize we were dealing with mortal students and not Olympian gods. This first pass at the new, integrated curriculum, this fiasco of absurd expectations by the faculty, served to hurt the propagation of the curriculum. Indeed, it was a lightning rod for critics, both students and faculty, and the sense of mistake never left me personally nor did it ever wash off the curriculum which is still available at Rose-Hulman. Since this incident, I have decided to plan for a new course in the following manner. Generate new ideas, create activities, build new visions, BUT only use a modest amount of the material on hand, be sensible in estimating what you believe students can accomplish, do not flood students, rather be gentle and reasonable in expectations.

Always listen to students, always keep a sense of the pace of activities in their lives, in your course - and, if necessary, change!

A few years ago two of my younger colleagues here at West Point had invested a great deal of effort into designing a wonderful project for our students in a course I was directing. One of the cadets emailed me with questions about plotting in MathCad. I opened the "Approved Solution" file on this project which my colleagues had prepared and cut out the appropriate code for just the plotting, not the answers to the project, mind you, just the plotting. I cleaned it up a bit, wrote some generic plotting advice issues on the page, pasted it into email, and fired it off, hoping this would help finesse presentation graphics issues for the cadet once she had accomplished the mathematics of the work. Little did I know that you cannot just send parts of MathCad in email – email sends the ENTIRE MathCad file. So what I had done was to send the cadet the entire solution to this huge class project. I learned later that night when I got a message that read, "Sir. Now that you have apparently sent me the entire solution to the project, I need guidance in how to proceed." I translated this into, "Look, you turkey, you sent the answer to the project questions to us [there were several to whom I sent it in response to similar queries]. Given our honor system and our unfair advantage this solution in hand gives us, what are we to do now? Oh, by the way, thanks a lot!"

When I discussed this with my younger colleagues who had worked so hard to develop the project I was embarrassed and I felt as though I had truly let them down. They were fully forgiving of my mistake and they advised we had only one recourse – send the entire solution to the project to all the cadets in the course. This meant we were going to deny them all the "doing" of the project. However, we wanted them to get something out of this experience, so we designed some questions as add-ons to the project. This is where I really learned from my mistake. Sure, taking care in sending email was something I learned, but wait 'til you here what I really learned from this mistake. I learned that our cadets really valued creative opportunities; they really meant it when they told us in course-end evaluations that they learned most by doing the projects. In doing the projects they saw things coming together which would otherwise not happen for them and this occurred only in the tough going of the project requirements. This I denied them with my mistake. These cadets, loudly and clearly, said they were disappointed in not being allowed to create their solutions, to build their own success, and to grapple with the hard issues of the project we had originally designed. In its place they recognized the weak, cosmetic questions we asked, based on the solution we denied them! They were not happy. My mistake in emailing the "small" MathCad material, which turned out to be the solution, made the cadets mad, not my colleagues! Most importantly, it gave me new insight into the student mindset and I really liked what I saw in them. I saw a willingness to accept challenge, to want to tackle difficult problems, and to be creative.

Purposeful mistakes – one someone else's and one all my own

There are mistakes that, if repeated, can serve you well. I give two examples. The first is a description of two overheads that I saw Ron Graham use. He is a distinguished researcher from the old Bell Labs, a world-class juggler, and a much sought-after speaker. When you set a transparency on the overhead projector you usually put your fingers over a section of

the edge to line it up, and then withdraw fingers after it is suitably set. What Graham did was to cut out a piece of paper in the shape of his fingers and tape this cutout to the edge of the transparency so that when he removed his hand and stepped back and proceeded to carry on with his speech his hand "stayed" on the overhead. Practically everyone in the audience did a double take and laughed. His "mistake" was leaving his hand behind! I have used this numerous times, in Kiwanis Club talks, classes, seminars, conference presentations, etc. It never fails to get a laugh, and there are times in ANY presentation when you need that, when you need a break, you need your audience to stop taking you and what you are about too seriously. The other slide Graham uses is one in which NO MATTER how he puts it up, or turns it, or flips it, there is something which reads backwards or upside down. You can produce such a slide using your copy machine, overheads and paste, or some computer software to reverse/invert some texts and images. He would try to respond, feverishly as one does when a slide is incorrect, to the calls of the crowd, "do this" "do that" etc., all to no avail, at which point he would simply chuck the overhead and say, "Well it probably was not important anyhow." But again his mistake was important, it served at some point in his presentation as an alerter, as a grabber, as a point of humor – something all of us can use now and then, be we presenter or listener.

Now for my own intentional mistake. When I was particularly anxious to be sure to announce a seminar to my students at the end of class I wrote the word "Seminar" on the upper right hand corner of the board. I put a box around it. However, in the fury of lecturing (I did a lot more of that in those days) I would erase the box and the word, "Seminar." So, I would pause and remount the reminder, by writing "Seminar" and boxing it in the upper right hand corner of the board. As I was in the fury of the last theorem being proved, I again erased the word and the box for the last of many times during the class. Then after the last "QED." I proceeded out the door. "Wait," the students said. "What about the seminar?" They were asking me about the seminar I was supposed to announce to them. They wanted to know what it was all about. So I told them. But they were very much aware of what was being erased and redone through out the class. It got their attention, it piqued their curiosity, and it gave me an idea. I would consciously make this mistake when I had such an announcement to make, e.g., throwing a pizza party for the class, a test(!), a class drop for a project, etc. By making the mistake, over and over, during the hour class, of erasing the message and re-doing the message I could get them to ask about the message. To this day I use this technique. This "mistake" works EVERY time.

Conclusion

I realize in reading these (and you may too) there is the "Duh" effect on some of them, but the important fact is that these mistakes are made by informed, rational beings, who are trying to do the right thing, but perhaps have a momentary, slipped link somewhere which permits bad things to happen. Perhaps the Situation is Normal and All Fouled Up (SNAFU) but it is not the end of the world, the situation can be remedied. Most importantly, it is an opportunity to learn, and the good news for me is that I have had lots of such learning opportunities, only a few of which I dared to share with you .

The M&M™ Lesson: A Cause of Heartburn for an Introductory Statistics Instructor

Captain Geoff Grobman
United States Air Force Academy

As part of my introductory statistics course we had finally arrived at a lesson I had been looking forward to all semester—confidence intervals. I anticipated this lesson because I could use candy M&Ms to teach the concept of confidence intervals to my students.

For this relatively common lesson plan, I handed out packets of M&Ms to each of my students and explained to them that each packet was a sample from the larger population of M&Ms. Next, I had them compute 95% confidence interval for blue M&Ms in the population of M&Ms based on their sample. Students then each visited the board and wrote down their upper and lower confidence limit. After this was completed I told them the actual population proportion of blue M&Ms. The lesson worked out wonderfully, since one out of my 20 students did not capture population proportion in his interval.

I left the classroom that day very excited about teaching. Several of my students spoke with me after class and said that they really enjoyed the lesson and it was the most fun they ever had doing math.

I started out the next class with the directed question to one of my students: "Explain in your own words what a 95% confidence interval is." His answer that 95% of all data falls in this interval took me aback. This answer was not only wrong, but it was completely wrong. I tried another student and received an equally poor response. I was really shocked by this. How could anyone not understand what a confidence interval was after such a wonderful lesson? I then asked one of my better students the same question and he responded with an acceptable answer. I was satisfied, but only slightly. I knew that this student would have given me a correct answer regardless of how the material was taught. Had my students forgotten what they learned the previous class? I became more disillusioned when a student of mine asked me after class, "What was the point of the whole M&M thing anyway?" The next exam confirmed my suspicion that a significant percentage of the class was still confused about the meaning of a confidence interval.

Upon further reflection, I came to the following conclusion. **I had mistaken my students' enthusiasm for learning**. The class enjoyed the M&M lesson greatly because it was a break from their routine and it gave them a chance to eat in class—a practice that is against regulations unless it is for academic purposes. Furthermore, they really enjoyed the interactive hands-on nature of the lesson. Therefore, I overestimated the pedagogical impact that the exercise would have on teaching the class what a confidence interval was.

Would I do the exercise again? Yes. I think that there was a subset of the class (I can't say how large) made up of visual/tactile learners who did experience a conceptual breakthrough because of this lesson. Additionally, the lesson provided a good break for the class from the monotony of introductory statistics. What would I change next time? I would lower my expectations on the impact of the exercise and make sure that I reinforce the concept using numerous approaches that match the varying learning styles in the class. For example, the cadet who asked me "what the point of the M&M thing was" happened to be someone who learned

well through nonvisual means. A more traditional teaching approach would have reached her better than the exercise.

In conclusion, I hope my story is not used to dispute the theory that student interest toward a lesson is necessary for learning to take place. However, enthusiasm alone is not a sufficient condition to ensure that learning takes place.

An Improbable Mistake

Professor V. Frederick Rickey
United States Military Academy

Years ago I was teaching a discrete mathematics course to a fairly large class, perhaps 45 students. We were doing counting arguments and the birthday problem came up. We worked out the problem, did some computations, and I remarked that in a class this big, surely there were two of us with the same birthday. So we decided to try it. I asked, who has a birthday in January? Lots of hands, but no matches. February? Again no matches. We proceeded through the months. The students grew more skeptical; I grew more apprehensive. Surely in this large a class we would get two people with the same birthday. When we got to November only one hand went up. So I called for December. Again there was only one hand. But I smiled and said "My birthday is in December, what date were you born?" She responded, the 17th. I said, hey, that's my birthday. There were boos and hoots and no one believed me. Proclaiming my innocence did no good. I had to get out my ID and pass it around.

My mistake was assuming the commutative law. I should have given my birthday and then asked the student for hers.

Build on Mistakes

Professor Donald Small
United States Military Academy

Building on mistakes holds more potential for long term learning than does building on correct solutions. The learning curve for building on mistakes is concave upward whereas the learning curve for building on correct solutions is often concave downward. It would be an unusual student who would what-if a correct solution and then use the solution as a base for exploration. On the other hand, correcting mistakes can be, with a little encouragement, an exploratory/discovery activity. Correct solutions provide immediate satisfaction, which can be displayed on bulletin boards and refrigerator doors. However unless this type of success has been filtered through a sequence of mistakes, it may have a short shelf life.

One of the most used processes in mathematics is built on analyzing mistakes. This is the iterative process of conjecturing and then modifying. This process has several names, including: "The Basic Approximation Process," "Guess and Check," and "Successive Approximations." In this process, a conjecture is made and then the error (the mistake part) is analyzed in order to develop a more accurate conjecture. Athletic coaches use this process as do music teachers, Drill Sergeants, cooks, and other people who are involved in learning or perfecting a skill, which includes all of us.

Does building on mistakes offer a "free ride" to students to do whatever they want because the results do not matter? Definitely not. Clear ground rules and expectations need to be clearly established at the start of a course. These would include

a. First time mistakes are expected and welcomed.
b. Repetition of a mistake is not acceptable.
c. All mistakes are to be revisited and analyzed.
d. All correct solutions are to be questioned.

The number and types of (first time) mistakes is one measure of student growth in the learning process. They are an indication of the willingness of a student to engage new ideas. The cliches "Nothing ventured, nothing gained" or "You can't win a race, if you don't enter" are representative of the advice given students to encourage them to become proactive in their learning. The number and types of mistakes a student makes is a measure of how well the student embraces this philosophy. Another important component in the learning process that is fed by mistakes is humility, a component that opens our thinking to other possibilities and encourages exploration.

While we strive for correctness and certainty, we must always question the result. We must guard against an illusion of success generated by obtaining correct solutions. We must also be ever alert not to let success generate a false pride that holds us back from trying for fear of making a mistake. Such a pride often manifests itself as a negative concavity in the learning curve. Some strategies, in addition to questioning a result, for avoiding these pitfalls are:

a. What-if the solved problem. What-ifing helps move the focus from just obtaining a correct solution to developing a conceptual understanding.
b. Explain the reasonableness of the solution.
c. Generalize the result.
d. Relate the solution process to a previous process.

If our ultimate goal is to create a learning environment that inspires exploration, that encourages questioning, and that leads to conceptual understanding, then we need to consider mistakes as positive learning opportunities. That is, we need to develop an environment in which mistakes are considered to be building blocks for learning.

Ralph Rodriguez: Teacher – An Appreciation

Father Gabriel B. Costa, Ph.D.
United States Military Academy

Before anyone ever heard of Jamie Escalante, there was Hoboken High School's Ralph Rodriguez. "Mr. Rod," as his students called him, was an engineer who became a world-class mathematics and physics teacher.

Raphael Antonio Rodriguez was born in Hoboken, New Jersey during the second decade of the last century. After graduating from high school in 1930, he went to New York University and received both bachelor's and masters degrees in mechanical engineering (in fact, he was ABD with regard to his doctorate). He married the former Lena Luciani, also of Hoboken. Mr. and Mrs. Rod (what else could she be called?) had two sons: Robert and Donald. After a number of years

working as an aeronautical engineer at the Curtiss-Wright corporation, he began taking education courses to better prepare himself for a new career: teaching.

And what a teacher he became! What a legacy he left behind! He was to become like a father to me; and, not only to me, but to a myriad of other students from the 50's, 60's and 70's.

Mr. Rod never stood on ceremony. His students always felt *comfortable* in classes. He used his *gift* of humor all the time. A humor that was warmly personal and extremely inviting…a disarming humor, not quite irreverent. And we always had a good laugh. Yet, the *line* between teacher and student *was never crossed*, because we respected Mr. Rod, as we did other teachers.

But Mr. Rod was different. His students also loved him.

I've been an educator for nearly thirty years, and each time I walk into the classroom, Mr. Rod comes with me. I remember not only what he taught, but also *how* he taught. He stressed simplicity. While I don't recall Mr. Rod quoting Einstein, he certainly believed that "everything should be as simple as possible; but not simpler". Whether he was teaching complex conjugates in Algebra II or the resolution of forces in Physics, he strove to be crystal-clear in his presentations. For example, he often began an explanation with "Now, the quadratic formula is *nothing more than*…". That frequently used and characteristically unpretentious tag immediately put his students at ease.

In the mid-1960's, calculus was not taught in our high school. Mr. Rod gave up his lunch hour to tutor interested students on the rudiments of this strange, new subject. It was *then* that I realized that I wanted to be *like* Mr. Rod. It was *then* that I realized that I wanted to be a mathematics *teacher*.

One morning, our homeroom teacher gave us the sad news that Mr. Rod had suffered a heart attack. That afternoon, I asked my mother to call Mrs. Rod. Mrs. Rod assured my mother that Mr. Rod would be fine, but that he had to take a temporary medical leave from teaching.

As soon as Mr. Rod returned home from the hospital, my friends (John DeSomma, Leo Smith, Julius Gottilla, Norby Machado, Karen Manzi, Frank Lenge, Ralph Terminiello, etc.) and I decided to visit Mr. Rod. We had to drive *all the way to* Wood-Ridge, NJ (that was far for us!). Nobody ever visited the home of a teacher in those days…and Leo wasn't even Mr. Rod's student! He was a football player who frequently cut "Study Hall" to sit in on Mr. Rod's classes. That's the way Mr. Rod was…a student *wanted* to be with him.

It surprised no one that even after a second heart attack, Mr. Rod was still teaching. He *had* to teach.

After I graduated from high school, I went to Stevens Institute of Technology, also in Hoboken. From time to time, I would visit Mr. Rod and actually teach a class every now and then (usually Geometry). Because of this experience, I started to become comfortable with public speaking…and the seeds of a life-long love affair with teaching were further implanted.

In the mid-1970's Mr. Rod retired to a family house in Cairo, New York with his beloved wife.

Mr. Rod and I never drifted apart. I would discuss my research with him… ask him questions about teaching…talk to him about anything. I would visit him when I could. On occasion, the "old gang" would come along: John (who became an engineer), Ralph (also an engineer), Julius

(teacher), and others. And, at times, we would speak about mathematics or mathematics education, but the real *continuing* education was in just being in his presence. He *never* lost his humor; he *never* lost his insights. We were, again, sitting at the feet of the Master.

I once asked him how he felt about his sons not going into a technical field such as mathematics or engineering. He replied that he just wanted them to be happy and healthy; but he also beamed when he mentioned that his granddaughter, Lisa, was in the process of getting a computer science degree. I like to think he considered me as a third son…the one who did go into mathematics. I hasten to add that he had many "third sons" and many "daughters".

Why did Mr. Rod have such an impact on his students? It's not difficult to explain.

Above all, Mr. Rod was *real.* Mr. Rod the teacher was in every way an extension of Mr. Rod the man. Allowing each student to be himself or herself, you knew that he was *concerned*…not just about the mathematics or the physics, but about the student *as an individual*. By his example, he showed us how *hard work* could really pay off. He taught us *not* to take ourselves too seriously and that laughter was an important *gift*. In his own way, he showed us the value of *family*…because he made us members of his family. His *passion* for teaching and mathematics spoke for itself. He taught *knowledge*; he imparted *wisdom.*

Mr. Rod was "extremely quotable". It wasn't just what he said; it was *how* he said it – his delivery, his timing, and his wit:

- "When confused, use the *Fanegal Factor*."
- "$E = mc^3$, for *slow* light"
- "DeSomma, you're asking what page we're on? In your case, any page."
- "Costa, your grades are so low that, from now on, you'll have to get *100* on every test, just to bring your final grade up to *0*."
- "Machado, you were young when Broadway was a prairie."
- "Manzi, don't thank God for your grade; thank Rod."
- "Terminiello, your board work is so bad, it isn't even wrong!"

More than anything, I wanted him to speak here at West Point. I wanted to *share* him with my colleagues and my students. I wanted all to *experience* this singular educator. Unfortunately, it never came to be.

Mr. Rod died a few years ago. It was my privilege to conduct his funeral Mass and to lay him to rest.

With all my heart, I believe that the Divine Mathematician has been laughing a little bit more since Mr. Rod got to Heaven.

Yet, Mr. Rod's work is not finished here on earth. Every time I walk into the classroom, Mr. Rod is with me.

And my students are much the better for it.

NOTE: The author wishes to thank Mr. John DeSomma, Mr. Julius Gottilla and Mr. Robert Rodriguez for their invaluable assistance and suggestions.

11th Annual Service Academy Student Mathematics Conference

Major David Bailey
United States Military Academy

On 13 April 2001, the United States Military Academy hosted the 11th Annual Service Academy Student Mathematics Conference. This annual opportunity for the mathematics and operations research majors at the federal service academies once again proved to be a huge success. Photographs of the presenters are included below.

A wide collection of talks, ranging from operations research and applied mathematics topics to pure and theoretical mathematics topics, were presented. In addition to the talks, the highlights of the conference included a history tour: "Mathematics on the Plain," given by Professor V. Frederick Rickey, USMA, and an impromptu tour of Quarters 100, the USMA Superintendent's quarters, graciously provided by Mrs. Christman.

USAFA: C1C Timothy Cook, C1C Deborah Herceg, C3C Frank Golf, and MAJ Scott Sears

USCGA: C1C Matthew Moyer, C1C Jon Berkshire, C1C Laura Millen, C1C Michael DiPace, C1C Jamie Pendergrass and LCDR Craig Swirbliss

USNA: M2C Katie Abdallah, M1C Daniel Post, M2C Benjamin Heineike, and LCDR Tony Fontana

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