Modernizing Calculus at the U.S. Air Force Academy

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Keywords:
Manuscript received on TBD; published on TBD.

Abstract:

This article discusses the calculus modernization initiative at the United States Air Force Academy (USAFA), which began in fall 2020 in response to guidance from the Mathematical Association of America, changes in science and technology, and direction from leadership in the United States Air Force (USAF) and United States Space Force (USSF). The calculus curriculum at USAFA has remained largely unchanged for decades, and many cadets struggle with the numeracy and algebra skills required to succeed in traditional college calculus courses; changes in technology have created opportunities to reimagine how calculus is taught. This articles describes the modernization process and argues that it provides cadets the needed skills to succeed academically, while emphasizing the importance of data literacy and acumen in countering near peer adversaries.

1 Introduction

The motivation for this article is the ongoing calculus modernization initiative at the United States Air Force Academy (USAFA). Starting in fall 2020, the Department of Mathematical Sciences (DFMS) at USAFA began to overhaul the calculus curriculum. This was in response to guidance from the Mathematical Association of America [11], dramatic advances in availability of and tools for analyzing data, and guidance and direction from leadership in the United States Air Force (USAF) and United States Space Force (USSF) [8]. Our work was partly informed by the successful Applied Calculus course developed by Dr. Danny Kaplan at Macalester College. His Applied Calculus course has been lauded as exemplary, especially regarding increasing STEM recruitment and retention. From fall 2020 through spring 2023, Dr. Kaplan served as the Ambassador Holland H. Coors' Chair at USAFA. In this capacity, he developed a textbook and accompanying software packages tailored for the USAFA calculus modernization effort [4].

In conducting this modernization effort, we aimed to provide an engaging first-year mathematics experience to serve the needs of cadets, client departments, and the United States Air Force (USAF) and United States Space Force (USSF). We identified three goals: 1) prepare cadets to be highly successful in the technical core and STEM majors, 2) improve

recruitment and retention in the STEM majors, and 3) better prepare future USAF/USSF officers for the skills they will be expected to have or acquire on active duty.

2 Background

The United States Air Force Academy is a fully accredited four-year undergraduate institution of higher learning. More than 1000 new cadets enter in each class year. All new cadets are required to complete (or gain credit for) two calculus courses within the first year. About 70% of each incoming class at USAFA are placed into Math 141 (Calculus I). These cadets complete their two-course requirement during their first year. The rest of the cadets place into Math 152 (Advanced Placement Calculus II), Math 253 (Advanced Placement Calculus III), or higher. Among those who are placed into Math 141, some have calculus experience, but the curriculum assumes no prior exposure.

The calculus curriculum at USAFA has remained largely unchanged for decades. However, many cadets find themselves without the requisite numeracy and algebra skills to succeed in a traditional college calculus course. As a result, like at other institutions across the country, calculus has served as a barrier to entry to STEM disciplines. This is not a new problem, and much work has been done to diagnose and treat the skills deficiencies prevalent among new undergraduates [1], and to address the issue at the high school level [7] [3]. Tragically, many of these cadets become discouraged because they struggle with skills that are not required in downstream STEM courses, or that are made obsolete by modern, accessible technology. Furthermore, STEM departments within the faculty have repeatedly noted that cadets who start in Math 141 are not prepared for downstream courses within their selected majors [9].

These issues are not unique to USAFA, and the Mathematical Association of America (MAA) has acknowledged this in their 2015 publication "A Common Vision for Undergraduate Mathematical Sciences Programs in 2025." In this article, the authors describe how changes in technology have created opportunities to reimagine how calculus is taught. They note that mathematical modeling is replacing many aspects of real-world implementation. Further, students can use technology for computations that are either unrealistic or burdensome to complete by hand. They point to seven separate studies that recommend "using technology to enhance student learning." [10] Institutions nation-wide have explored ways to adapt to the situation while shifting the focus on broader skills like quantitative literacy [5]. Some have even proposed making this shift earlier, at the high school level [12] [2].

Simultaneously, leadership in the Department of Defense (DoD), USAF and USSF, have emphasized the importance of data literacy and acumen in countering near peer adversaries [8]. Regardless of academic background and choice of career, future officers will be expected to have some familiarity with data. Additionally, familiarity with tools and methods for analysis and interpretation of that data is also expected. In response to this emphasis, USAFA has recently instituted a data science major. Adapting core calculus is a way to capture most cadets at USAFA.

3 Goals of Modernization

There are three goals we hoped to achieve in modernizing our calculus curriculum. These goals helped drive the choices we made in selecting topics, materials, and methods for our courses. We also conducted assessment with these goals in mind. Ultimately, we aim to provide an engaging first mathematics experience to serve the needs of cadets, client departments, and the USAF and USSF.

3.1 Goal 1: Preparation for Technical Core and STEM Majors

First, we acknowledge that calculus is a fundamental branch of mathematics that plays a crucial role in various technical and scientific fields. However, the traditional way of teaching calculus has been criticized for being overly abstract, outdated, and disconnected from modern applications. To prepare cadets for technical courses and STEM majors, we hope to integrate innovative approaches and technologies that emphasize the relevance, utility, and interdisciplinary nature of calculus. This includes using real-world examples, modeling, and visualization tools to motivate cadets and enhance their conceptual understanding, problem-solving skills, and critical thinking abilities. By modernizing calculus, our cadets can gain the skills and confidence needed to tackle complex problems in engineering, physics, economics, and other STEM fields, and become better equipped to thrive in the digital age.

3.2 Goal 2: Improve STEM Recruitment and Retention

Tied closely to our first goal, we hope that by making the subject more engaging and accessible, we will encourage more cadets to declare STEM majors. By focusing on modeling and real-world applications, we aim to increase student interest and motivation in STEM subjects. Also, by demonstrating the interdisciplinary nature of calculus, we can attract a more diverse group of students to STEM fields. In short, by modernizing calculus, we hope to build a stronger pipeline of future STEM professionals who are well-prepared for the challenges of the 21st century.

3.3 Goal 3: Prepare USAF/USSF Officers

At its core, the U.S. Air Force Academy is responsible for developing the next generation of leaders needed to serve in tomorrow's armed forces. In 2020, the Department of Defense released a Data Strategy document stating that the "DoD is a data-centric organization that uses data at speed and scale for operational advantage and increased efficiency." [8] Regardless of their intended academic major or career, all cadets need to be prepared to enter such a data-centric organization. A modernized calculus curriculum can help prepare cadets for the future by introducing the necessary tools and skills to solve complex problems and think critically in a rapidly changing technological landscape.

4 In-class Implementation

The modernized version of the calculus curriculum deemphasized some traditional topics, making room for more modern material. Topics that were added or emphasized include mathematical modeling, multivariable functions, introduction to the R programming language, dimensional analysis, linear algebra, ordinary differential equations, and an emphasis on conceptual understanding of derivatives and accumulation. Topics that were deemphasized or eliminated include formal treatment of limits, formal treatment of limit definition of a derivative, implicit differentiation, l'Hopital's Rule, related rates, and series and sequences. Cadets are taught how to use R (through the "mosaic" and "mosaicCalc" packages) to define and visualize functions and data, model data with a function, find the derivative or partial derivative of a function, and to calculate definite integrals. However, cadets are still expected to demonstrate competency in fundamental calculus skills (basic differentiation/integration skills such as chain rule, product rule, integration by substitution and integration by parts) without the aid of a computer. The sequence was divided into six blocks (three per semester) with the following topics.

1. Functions and Modeling
2. Linear Algebra
3. Derivatives
4. Optimization
5. Integration
6. Ordinary Differential Equations

We underwent a phased enrollment scheme for the modernized calculus sequence. In the initial run (AY 20-21), only 150 cadets were enrolled while the remainder took our traditional calculus sequence. The goals of this initial run were to demonstrate feasibility and to prepare to scale the course for larger offerings. The following year, about 400 cadets were enrolled in the modernized version of calculus, roughly half of all cadets placed into Math 141 (Calculus I). The goals of this second run were to demonstrate efficacy on a larger scale and to prepare the course for full enrollment. This larger run allowed us to stabilize the curriculum, develop training for our new instructors, and to address large-scale administration issues. Finally, during AY 22-23, all cadets who placed into Math 141 took the modernized version. Total enrollment during that year was about 675.

At the beginning of this process (AY 20-21), we used the textbook "Mathematical Modeling and Applied Calculus" by Joel Kilty & Alex M. McAllister [6]. Late in the first year of execution, we used a free online textbook built by Dr. Kaplan, our Coors Chair [4] The intent of this text was to build off of what Kilty and McAllister had developed, but customized for our own curriculum. We used this online text exclusively during AY 21-22. We found that while we appreciated the customized nature of the textbook, it was not quite ready for use in the expanded offering. For AY 22-23, we returned to the Kilty & McAllister text while we focused on course stability and long-term sustainability. While Kilty & McAllister does not perfectly fit the needs of our sequence, it serves as the best alternative while we consider our textbook options.

The phased enrollment of modernized calculus unfortunately coincided with the COVID-19 pandemic. This presented many challenges. During the first year (AY 20-21), the course met largely over Microsoft Teams, synchronously. In AY 21-22, all lessons

were delivered in person, but sometimes in combined classes. The standard class size of a calculus class at USAFA is about 18-22 cadets. Sometimes during AY 21-22, multiple sections were combined into a small lecture hall, with 60-80 cadets in attendance at once. Combining sections was meant to help train new instructors in the modernized calculus approach. However, the larger class sizes hindered participation. Overall, we found that the drawbacks of this hybrid format outweighed the benefits. In AY 22-23, we returned entirely to traditional, small-section delivery.

As mentioned earlier, the modernized version of calculus incorporated the R programming language. In the first year of execution, we utilized a web-based "sandbox" environment. This approach had benefits and drawbacks. The sandbox allowed cadets to use the R tools presented without needing to install any software on their computers. This meant that cadets could focus on analysis rather than other issues such as importing data, version control, installing/loading packages, etc. However, when we expanded the course in the second year, we found that the server where we hosted the sandbox was not capable of handling the increased demand. Furthermore, it was difficult for cadets to access documentation for R functions. Mid-way during AY 21-22, we required cadets to install RStudio locally and install all relevant packages. After the initial installation was complete, we found it to be simpler to use RStudio locally rather than the sandbox. Local RStudio is also more representative of how the R programming language is used, so better prepares cadets to use R in the future.

In AY 22-23, cadets were required to locally install R and RStudio. Furthermore, because we readopted the Kilty & McAllister text, use of R was simpler for the cadets. The Kilty & McAllister text incorporates all R commands in the text, so it served as the primary resource for using R.

5 Assessment

In order to evaluate the effectiveness of the modernization program, we developed an assessment plan with three focus areas tied to the goals mentioned above:

1. Fundamental Skills/Competencies
2. STEM Retention
3. Performance in Downstream Courses

5.1 Focus Area 1: Fundamental Skills/Competencies

In this focus area, we assessed how cadets performed in each version of calculus with respect to understanding of calculus topics. Cadets were assessed in estimating and obtaining derivatives and antiderivatives analytically (without the use of software) and in interpreting derivatives and antiderivatives. We asked a series of common questions between the two versions. These included basic derivatives and antiderivatives, graphical approximation of derivatives and antiderivatives, and interpretation of derivatives, antiderivatives and definite integrals. We made comparisons based on two metrics: average score in each topic and percent deficient in each topic. These comparisons were made

only during the AY 21-22 offering, as that is when we had a roughly even enrollment between the two versions of calculus.

Results: Overall, cadets in the modernized version of calculus had slightly higher mean scores (69.19% vs 68.11%) and a slightly lower percent of deficient scores (49.35% vs 50.48%). Note that we define "deficient" as carrying a grade of below 70%. These differences were marginal and depended on the types of questions. For example, cadets in the modernized version of calculus performed slightly worse on mechanical skills such as integration by parts problems, but better on problems evaluating conceptual understanding (interpretation of differentiation and the definite integral).

Interpretation/Action: Cadets in the modernized version of calculus covered differentiation during Block 2 and integration in Block 3 (both during first semester). These blocks were designed to focus more on applications and less on fundamental skills. Cadets were taught to use RStudio to find derivatives and antiderivatives, but were assessed on the exams without access to RStudio.

As a result, we concluded that the AY 21-22 modernized calculus version was less conducive to retention of fundamental skills (basic differentiation/integration). The syllabus only allowed for some in-class guided practice in these skills (four lessons for differentiation in Block 2 and four lessons for antiderivatives in Block 3). Some of this practice took place in a lectinar format, and cadets did not receive the individual feedback they would get in a smaller classroom environment. Further, accumulation/integration was presented immediately after differentiation. Some cadets found it difficult to fully grasp differentiation prior to moving directly into antiderivatives. Regardless, cadets in the modernized version performed similarly to those who received a more traditional treatment (scores in both courses were low).

While we acknowledge that part of the spirit of calculus modernization is a de-emphasis on rote skills easily performed by a computer, we concluded that it was important to add a little more focus on these skills. Based on this assessment, we adjusted the curriculum for AY 22-23. In the AY 22-23 modernized version, these two topics were split into different semesters: differentiation is covered in Block 3 during Math 141 and integration/accumulation is covered in Block 5 during Math 142 (Calculus II). Furthermore, we added four additional lessons dedicated to fundamental derivative/antiderivative topics (chain rule, product rule, integration by substitution/parts). Finally, we returned exclusively to the small classroom environment (no meetings in lectinars).

5.2 Focus Area 2: STEM Recruitment and Retention

One of the goals of modernizing the calculus curriculum is to improve STEM recruitment and retention. We believe that a calculus course that uses modern tools and focuses more on modeling real-world problems will encourage cadets to pursue technical majors. To evaluate this, we compared percentages of cadets who declared STEM majors for the two versions of calculus. Additionally, we tracked and compared rates of retention in STEM majors. For the purpose of this assessment, STEM majors include: Aeronautical Engineering, Astronautical Engineering, Biochemistry, Biology, Chemistry, Computer Science, Computer Engineering, Data Science, Economics, Electrical Engineering, Mathematical Sciences, Mechanical Engineering, Meteorology, Operations Research, Physics, Systems

Engineering, Space Operations.

Results: In class year 2025, cadets in the modernized version tended to declare STEM majors at a similar percentage to those from the traditional version. This was a reversal of what we saw among cadets in class year 2024. STEM declaration rate across the class of 2024 was 58% and for class year 2025, it was about 61%. Among those that started in the modernized version, STEM declaration rate increased from 39% in class year 2024 to 51% in class year 2025. Among those who started in traditional calculus, these values were 44% and 51%.

Interpretation/Action: For class year 2025, we saw STEM declaration rate for cadets in modernized version match up with those who started in the traditional version. As this curriculum matures, we expect these numbers to improve. We have successfully integrated concepts from other fields such as economics and electrical engineering. We will continue to reach out to other departments in order to find contextual examples that tie our approaches in modernized calculus to majors our cadets are considering declaring. Furthermore, in AY 22-23, two of our instructors have been from other departments (the Department Electrical and Computer Engineering and the Department of Physics). Their experiences within the modernized version will help us improve and customize our curriculum for our cadets.

5.3 Focus Area 3: Performance in Downstream Courses

Another goal of modernizing the calculus curriculum is to better prepare cadets for success in subsequent technical courses. To this end, we compared performance and deficiency rates in follow-on (and concurrent) courses between those who took the modernized versions and those who took the traditional versions. Follow-on and concurrent courses include (but are not limited to): Physics, Computer Science, Calculus III, Differential Equations, Chemistry, and Mechanical Engineering. We recorded average performance (grade point average) of cadets from each course in the above listed areas, as well as percentage of cadets from each cohort with deficient grades in the above listed courses.

Results: Overall, cadets who started in the modernized version performed no better or worse than those who started in the traditional version. In the most recent semester for which we have data (Spring 2022), cadets who started in modernized calculus had an average grade point average (GPA) of 2.603 in STEM courses (on a 4.0 scale). Cadets who started in traditional calculus had an average GPA of 2.653. These differences were small and varied by course. In some courses (such as Computer Science) cadets who started in the modernized version performed better while in others (such as Mechanical Engineering), they performed worse. Again, these differences were minimal and paled in comparison to the differences between cadets who started in Math 141 and Math 152 (Advanced Placement Calculus II). These cadets had an average GPA of 3.671 in STEM courses.

Interpretation/Action: We are optimistic about the results we have seen in this Focus Area. Considering the relative immaturity of this course, we are pleased that performance in downstream STEM courses remains largely unchanged thus far. As the course matures, and as we respond to some of the feedback we hear from other departments and within our own department, we expect such downstream performance to improve among cadets

enrolled in the modernized version of calculus. For example, as in Focus Area 1, we recognize that while a modernized version of calculus de-emphasizes rote mechanical skills, some downstream courses in Physics and Mechanical Engineering assume proficiency in those skills. Thus, we adapted our curriculum for AY 22-23 syllabus as discussed in Focus Area 1.

5.4 Assessment Summary

Overall, we are encouraged by the assessment of the modernization effort thus far. STEM declaration rates are improving for the modernized version of calculus. Performance in calculus topics and in downstream technical courses is roughly equivalent between the two versions of calculus, and we have adjusted our curriculum in response to lessons learned from the first two years of implementation. Given the immaturity of the original offerings and the administrative difficulties presented by COVID, we are encouraged by the results thus far. As the sequence becomes more solidified and sustainable, we expect to see substantial improvement.

6 Future Effort and Analysis

Our calculus modernization effort is an ongoing process that requires constant evaluation and improvement. We continue to look for ways to improve by seeking feedback from our cadets and colleagues in client departments, as well as by leveraging our internal department assessment process. Additionally, we will continue to explore new technologies, course materials, and pedagogical techniques that can enhance the learning experience for students. For example, we currently use the R software package and the Kilty & McAllister textbook. However, as our course matures, and we collaborate with other institutions that modernize their own calculus curricula, we may find other materials that better meet the needs of our cadets and faculty.

Additionally, we will continue to monitor progress of cadets during and after enrollment in the modernized version of calculus. Starting in AY 22-23, we no longer have the opportunity for side-by-side comparison between the two version of calculus. However, we will continue to collect information regarding fundamental skills, conceptual understanding, STEM recruitment and retention, and performance in downstream technical courses. Doing so will allow us to watch longitudinal trends as the modernized courses mature.

7 Conclusion

Over these last three years, we (the Department of Mathematical Sciences) have found that the modernization of our calculus curriculum can bring significant benefits to our cadets. By incorporating technology, modeling of data, and real-world applications, we aim for our modern calculus courses to become more engaging and relevant. We are hopeful that cadets that experience this curriculum will be better equipped to apply calculus concepts to real-world problems, and will be better prepared to meet the challenges in the USAF

and USSF of the future. This modernization effort is an ongoing process that requires continued evaluation and improvement. As technology and pedagogy continue to evolve, it is important to remain open to new approaches and feedback from cadets and faculty to ensure that calculus education continues to meet the changing needs of our future officers in the 21st century.

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