

I. Introduction

Fantasy Basketball ranks secondly behind Fantasy Football for fantasy sports popularity (Statistica 2018), but trails substantially behind in total participation. The relatively small participation could ostensibly explain the dearth of research and analysis dedicated to the game. That explanation is refuted, however, by Fantasy Baseball which ranks third yet is the seminal fantasy game with a litany of historical analyses from the pioneers of fantasy sports. Within this void is a lack of academic rigor the other fantasy sports enjoy. Fantasy basketball enthusiasts and analysts are missing an opportunity to exploit a similar environment to collect, analyze, and assess game data. Fantasy basketball websites, nevertheless, try to fill in the gap in demand and cater to fantasy basketball devotees. These sites employ colloquially named “fantasy experts” who create and build player rankings with different qualitative and quantitative techniques to provide their audience a way to select players for their fantasy teams.

Fantasy experts employ a host of techniques to quantify their rankings of a player’s value before the season starts (CBS Sports Staff 2018). Top fantasy basketball sites ESPN and FantasyPros employ a player ranking system that simply takes a player’s average statistics from the previous season, then standardizes and aggregates them against other players to create a comprehensive “z-score” for a player. Ranking the z-scores in ascending order creates the basis for nascent subjective evaluation and is sufficient for most blasé fans. Competitive fans, however, find these rankings rudimentary and not granular enough to capture the idiosyncrasies of the game. Correspondingly, fantasy experts apply subjective criteria to bridge this gap and incorporate what they observe on the hardwood to anticipated player performance in fantasy. Casual observation, however, indicates that experts are victims of their own biases and thus tend to favorably rank popular players over those with similar capabilities. Resultantly, the top experts’ picks include disproportionately more mature and vetted players, skewing the rankings and undervaluing less recognized players.

This paper seeks a more effective fantasy basketball player ranking. The decision making process should include objective and subjective factors, balanced heuristics, and a priori assessments. While the portfolio of Multi Criteria Decision Making (MCDM) all differentiate and evaluate between selected criterions the quantification of a user’s subjectivity is paramount. One such method, developed by Hwang and Yoon (1981) is aptly named the “Technique for Order Preference by Similarity to an Ideal Solution”, or TOPSIS. This method posits the best choice will have the shortest geometric distance to the most positive ideal solution and be the furthest away from the least ideal, or negative, solution. Chen and Hwang (1992) expanded the methodology to include the decision maker’s subjective weighting criteria, incorporated up front, to create a hierarchal structure of the data being analyzed. This paper utilizes Chen and Hwang’s methodology, referred to as the classical TOPSIS technique (Roszkowska 2011).

Description of the problem.

Seasoned fantasy managers recognize they need every advantage to beat their competition, beginning with fantasy draft where they face decisions about the same talent pool. Knowing which players to draft is as important as knowing when to draft them. Most novice

players do not conduct their own analysis and rely on fantasy expert guides which recommend whom to draft and when. These aggregated recommendations are called average draft positions, or ADP. By anticipating novice managers will over-rely on these recommendations, the skilled manager can “abuse the ADP” (Hribar 2018) by exploiting the inefficient rankings with their superior custom approach and maximizing their return on investment.

FantasyPros.com, a popular fantasy sports website, aggregates, averages, and adjudicates multiple fantasy basketball experts’ rankings to produce a fantasy players’ overall ranking. These rankings serve as the competitive exemplar for this paper’s TOPSIS rankings. The individual experts’ exact methodologies are ambiguous. General observations reveal a mix of bottom-up and top-down statistics, team trends and patterns, and qualitative assessments to shape their projected rankings. After the first few picks, subjectivity appears to dominate the experts’ rankings. The site enforces ranking quality based on the experts’ ability to project player rankings jettisoning those that deviate too far away from actual player performance. An example of FantasyPros.com’s rankings are shown below.

RANK	PLAYER	BEST	WORST	AVG	STD DEV	ADP	VS. ADP
1	Anthony Davis (LAL - PF,C)	1	4	1.3	0.8	1	+0
2	James Harden (HOU - PG,SG)	1	5	2.3	1.1	3	+1
3	Giannis Antetokounmpo (MIL - SF,PF)	2	7	4.0	1.5	2	-1
4	Karl-Anthony Towns (MIN - C)	2	8	4.3	1.9	5	+1
5	Kevin Durant (BKN - SF,PF) OUT	4	9	5.1	1.2	6	+1
6	LeBron James (LAL - SF,PF)	2	11	6.1	2.4	4	-2
7	Stephen Curry (GSW - PG,SG)	3	10	6.6	1.9	8	+1
8	Nikola Jokic (DEN - PF,C)	5	23	8.9	4.1	9	+1
9	Damian Lillard (POR - PG)	8	15	10.7	1.9	10	+1

Figure 1 Illustrative example of FantasyPros’ rankings.

Introduced earlier, one tool FantasyPros.com experts exercise is the z-score. As a standalone metric, the z-score’s merit is standardization of a player’s statistics against other players, allowing for easy comparison with a mean of zero and standard deviation of one. When comparisons are required amongst several criterion analysts must scale the data to allow for like-wise comparisons (Wiesen 2006). As an example, fantasy basketball requires multivariate evaluation between the number of points a player scores per game (PPG) and their Free Throw Shooting Percentage (FT%). Fundamentally, it would be problematic to compare the means of the two metrics together as they have different statistical properties (e.g. sample size). Standardizing allows for a linear comparison between the scoring metrics and also serves to identify outliers.

The principal drawback of the FantasyPros.com expert rankings is their reliance on subjective evaluation of the z-score after calculation. It is observed that the experts insert subjectivity into their rankings by adjusting player rankings based on their inductive assessments, which varies significantly beyond the top rankings. Conversely, the indiscriminate

summing and aggregation of a player's individual z-scores strip the value of the standardized statistics. Specifically, the standard deviation of a statistic from an exceptional player loses its interval value due to a lack of weighting. The value of weighting these standardized statistics is to reintroduce the interval, or tiering, of especially scarce or over-abundant statistics. If a player is the league leader in a category then they should be rewarded based on this performance with an increase that is proportionate to the statistic's availability. Inversely, if a player performs poorly in a statistical category then the ranking should penalize him accordingly. The hypothesis, then, is that fantasy basketball rankings created through TOPSIS will outperform the subjective rankings from FantasyPros.com.

II. Mathematical Basis and Computational Procedure for TOPSIS

The classic TOPSIS procedure, developed by Chen and Hwang (1992) is a seven-step process that starts with the construction of a decision matrix from the data source and ends with the ranking by closest ideal solution for each attribute. The Euclidean distance approach will find the shortest distance to the positive ideal solution and the furthest distance from the negative ideal solution. The seven steps are listed below and will be applied in the following section.

Step 1. Construct the decision matrix and determine (if applicable) the weight of the criteria.

The construction of the decision matrix, usually in a table, is the collection of all alternatives (M) with each respective attribute (N). The matrix, $X=(x_{ij})$, therefore represents this collection where x_{ij} refers to the property of the i-th alternative of the j-th attribute.

The weighting criteria may be qualitative or quantitative depending on the decision maker. The quantitative weighting methodology employed in this paper is the normalization of each criteria's Pearson's Coefficient of Skewness #2. This descriptive statistic provides an objective technique to determine the skewness (positive, negative, symmetric) of each criteria and thus indicating its relative scarcity. The coefficients are determined by subtracting the mean from the median, multiplying by three, and dividing by the standard deviation as indicated:

$$Sk_2 = \frac{3(\bar{x} - \tilde{x})}{\sigma} \text{ Equation 1}$$

The coefficients for each of the criteria is aggregated and the sum calculated. The overall weighting should add up to 1 so each criteria is subsequently divided by the aggregated sum and finally represented as w_j . If all attributes are equally important then no weighting criteria may be necessary.

Step 2. Calculate the normalized decision matrix.

The transformation of the attributes from their natural scales into non-dimensional attributes allows for the comparison amongst each other. Because the units of measure (e.g. PPG vs. FT%) is not directly comparable, this normalization process is necessary. Each attribute then

follows a normal distribution and falls on a scale of 0 to 1. The matrix X, is normalized using this formula for each attribute:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \text{ Equation 2}$$

Step 3. Calculate the weighted normalized decision matrix.

If the attributes are equitable with no weighting needed this step is skipped. However, if the attributes were given a determined weight (w_j) from Step 1, the normalized decision matrix, X, developed in Step 2 must will be transformed to a Weighted Normalized Decision Matrix. This is achieved by multiplying each normalized weight (w_j) by the normalized attribute (n_{ij}) thus creating a weighted normalized value v_{ij} .

$$v_{ij} = w_j n_{ij} \text{ Equation 3}$$

Step 4. Determine the positive and negative ideal solutions.

The positive and negative ideal solution for each attribute is the respective desired or undesired maximum and minimum of each of the attributes. Specifically, the positive ideal solution (A^+) is the absolute maximum of the attribute that maximizes the benefit. The negative ideal solution (A^-) is the opposite and minimizes the benefit. As such, A^+ and A^- can be determined as such with J representing the benefit of attribute N:

$$A^+ = \{max v_{ij} | j = J\} = \{v_1^+, v_2^+, \dots \dots \dots, v_n^+\} \text{ Equation 4}$$

$$A^- = \{min v_{ij} | j = J\} = \{v_1^-, v_2^-, \dots \dots \dots, v_n^-\} \text{ Equation 5}$$

Step 5. Calculate the separation measures from the positive and negative ideal solution.

The separation from the positive and negative ideal solution is determined by finding the one-dimensional Euclidean distance of each weighted normalized value (v_{ij}) from each of the attribute’s ideal solutions (v_j^+, v_j^-) thus determining each geometric mean. The calculated distance is fittingly defined as d, with d^+ representing the separation between the positive ideal solution and d^- representing the negative ideal solution. Within the matrix, the Euclidean distance is simply the distance between two points on a line with the positive and negative ideal solutions set as the point of origin. Therefore, the separation distances are calculated as:

$$d_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}, i=1,2,\dots\dots\dots m \text{ Equation 6}$$

$$d_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, i=1,2,\dots\dots\dots m \text{ Equation 7}$$

Step 6. Calculate the relative closeness to the positive ideal solution.

The relative closeness of an alternative to the ideal solution can be determined through the application of the following formula:

$$C_i = \frac{d_i^-}{d_i^- + d_i^+}, 0 < C_i < 1, i = 1, 2, \dots, m \text{ Equation 8}$$

As the parameters help indicate, the best alternative is calculated closest to 1.

Step 7. Aggregate and rank the preference order.

All of the alternatives are now aggregated, summed, and ranked in descending order based on the value of each respective C_i . As stated in Step 6, the closer the alternative is to 1 the closer it is to the positive ideal solution.

III. Application of TOPSIS

To perform the steps listed in the methodology above, 595 individual entries of player data from the 2016-2017 NBA season were scraped from BasketballReference.com, an online data repository. The raw data was cleaned, had duplicates removed, and trimmed to replicate the most popular format in fantasy basketball, the 9-Category Rotisserie League Settings¹. The breakdown of these nine data fields are: (1) Points per game (PPG), (2) Total Rebounds (RBS), (3) Blocks (BLK), (4) Steals (STL), (5) Turnovers (TOV), (6) Assists (AST), (7) Three-pointers Made (3PM), (8) Free Throw Percentage (FT%), and (9) Field Goal Percentage (FG%). The metrics are all quantitative. And with the exception of Turnovers, higher counts are better. A snapshot of the data can be seen in the figure below. The data cleaning leaves 363 unique samples for further transition and analysis.

Player	G	FG	FGA	FG%	3P	3PA	3P%	2P	2PA	2P%	eFG%	FT	FTA	FT%	ORB	DRB	TRB	AST	STL	BLK	TOV	PF	PTS	PPG
Aaron Brooks	65	121	300	0.403	48	128	0.375	73	172	0.424	0.483	32	40	0.8	18	51	69	125	25	9	66	93	322	4.95
Aaron Gordon	80	393	865	0.454	77	267	0.288	316	598	0.528	0.499	156	217	0.719	116	289	405	150	65	40	89	172	1019	12.74
Al Horford	68	379	801	0.473	86	242	0.355	293	559	0.524	0.527	108	135	0.8	95	370	465	337	52	86	115	138	952	14.00
Al Jefferson	66	235	471	0.499	0	1	0	235	470	0.5	0.499	65	85	0.765	75	203	278	57	19	16	33	125	535	8.11
Alan Williams	47	138	267	0.517	0	1	0	138	266	0.519	0.517	70	112	0.625	94	198	292	23	27	32	37	125	346	7.36
Alec Burks	42	99	248	0.399	25	76	0.329	74	172	0.43	0.45	60	78	0.769	17	103	120	30	18	5	35	50	283	6.74
Alex Abrines	68	134	341	0.393	94	247	0.381	40	94	0.426	0.531	44	49	0.898	18	68	86	40	37	8	33	114	406	5.97
Alex Len	77	230	463	0.497	3	12	0.25	227	451	0.503	0.5	150	208	0.721	156	354	510	44	38	98	102	242	613	7.96
Alexis Ajinca	39	89	178	0.5	0	4	0	89	174	0.511	0.5	29	40	0.725	46	131	177	12	20	22	31	77	207	5.31
Al-Farouq Aminu	61	183	466	0.393	70	212	0.33	113	254	0.445	0.468	96	136	0.706	77	374	451	99	60	44	94	102	532	8.72
Allen Crabbe	79	303	647	0.468	134	302	0.444	169	345	0.49	0.572	105	124	0.847	19	207	226	94	54	20	62	171	845	10.70
Amir Johnson	80	213	370	0.576	27	66	0.409	186	304	0.612	0.612	67	100	0.67	118	248	366	140	51	62	77	211	520	6.50
Andre Drummond	81	483	911	0.53	2	7	0.286	481	904	0.532	0.531	137	355	0.386	345	770	1115	90	124	89	152	237	1105	13.64
Andre Iguodala	76	219	415	0.528	64	177	0.362	155	238	0.651	0.605	72	102	0.706	51	253	304	261	76	39	58	97	574	7.55
Andre Roberson	79	215	463	0.464	45	184	0.245	170	279	0.609	0.513	47	111	0.423	98	305	403	79	94	79	51	204	522	6.61
Andrew Harrison	72	117	360	0.325	43	156	0.276	74	204	0.363	0.385	148	194	0.763	23	113	136	198	53	21	85	194	425	5.90
Andrew Wiggins	82	709	1570	0.452	103	289	0.356	606	1281	0.473	0.484	412	542	0.76	102	226	328	189	82	30	187	183	1933	23.57
Anthony Davis	75	770	1526	0.505	40	134	0.299	730	1392	0.524	0.518	519	647	0.802	172	712	884	157	94	167	181	168	2099	27.99

Figure 2 Cleaned 2016-2017 NBA Data for Analysis

Applying the methodology, the cleaned data becomes the matrix, X, with M alternatives, or players, and N attributes. The nine statistical categories are transformed from season totals to “per game averages”. This matrix is expressed below and what TOPSIS will be applied to.

¹ Rotisserie settings are the aggregate counting of each statistical category and assigning a rank order to the ascending counts. The player with the greatest sum of the counts is the winner.

Player	FG%	3PM/G	FT%	TRB/G	AST/G	STL/G	BLK/G	TOV/G	PPG
Aaron Brooks	0.403	0.738	0.8	1.061538	1.923077	0.384615	0.138462	1.015385	4.95
Aaron Gordon	0.454	0.963	0.719	5.0625	1.875	0.8125	0.5	1.1125	12.74
Al Horford	0.473	1.265	0.8	6.838235	4.955882	0.764706	1.264706	1.691176	14.00
Al Jefferson	0.499	0.000	0.765	4.212121	0.863636	0.287879	0.242424	0.5	8.11
Alan Williams	0.517	0.000	0.625	6.212766	0.489362	0.574468	0.680851	0.787234	7.36
Alec Burks	0.399	0.595	0.769	2.857143	0.714286	0.428571	0.119048	0.833333	6.74
Alex Abrines	0.393	1.382	0.898	1.264706	0.588235	0.544118	0.117647	0.485294	5.97
Alex Len	0.497	0.039	0.721	6.623377	0.571429	0.493506	1.272727	1.324675	7.96
Alexis Ajinca	0.5	0.000	0.725	4.538462	0.307692	0.512821	0.564103	0.794872	5.31
Al-Farouq Aminu	0.393	1.148	0.706	7.393443	1.622951	0.983607	0.721311	1.540984	8.72
Allen Crabbe	0.468	1.696	0.847	2.860759	1.189873	0.683544	0.253165	0.78481	10.70
Amir Johnson	0.576	0.338	0.67	4.575	1.75	0.6375	0.775	0.9625	6.50
Andre Drummond	0.53	0.025	0.386	13.76543	1.111111	1.530864	1.098765	1.876543	13.64
Andre Iguodala	0.528	0.842	0.706	4	3.434211	1	0.513158	0.763158	7.55
Andre Roberson	0.464	0.570	0.423	5.101266	1	1.189873	1	0.64557	6.61
Andrew Harrison	0.325	0.597	0.763	1.888889	2.75	0.736111	0.291667	1.180556	5.90
Andrew Wiggins	0.452	1.256	0.76	4	2.304878	1	0.365854	2.280488	23.57
Anthony Davis	0.505	0.533	0.802	11.78667	2.093333	1.253333	2.226667	2.413333	27.99

Figure 3 Final Matrix before TOPSIS applied

In his book “Assessment, Measurement, and Prediction for Personnel Decisions” Mr. Robert Guion (2011) stated that “a weighting method should be based on rational, theoretical grounds rather than on computations alone.” The value of a particular player attribute is related to how scarce the attribute is among the pool. Scarcity can be defined by the unlikelihood that a random player can achieve a meaningful statistical threshold. Thus, a rational approach would be to weight scarce attributes more than abundant attributes. Scarcity can be quantified based on the foundation of normality testing. As an exemplar, in the figure below, steals are observed to follow a slight, positive skew from a symmetric distribution. The average number of steals are 0.8 STL/G with a standard deviation of plus or minus 0.4 steals. The maximum number of steals are slightly above 2 indicating that though there are elite steal producers, they do not perform far above the median. As a result steals should not be weighted greater compared to other statistics. Comparatively, the average number of blocks is 0.5 plus or minus 0.4 BLK/G with a maximum of 2.6. The majority of the players do not register more than 0.5 BLK/G (194/305 or 64%). But the density curve below shows a heavier, positive skew. This means that elite shot blockers are scarce and make a substantial performance difference. This Player attribute should be weighted accordingly.

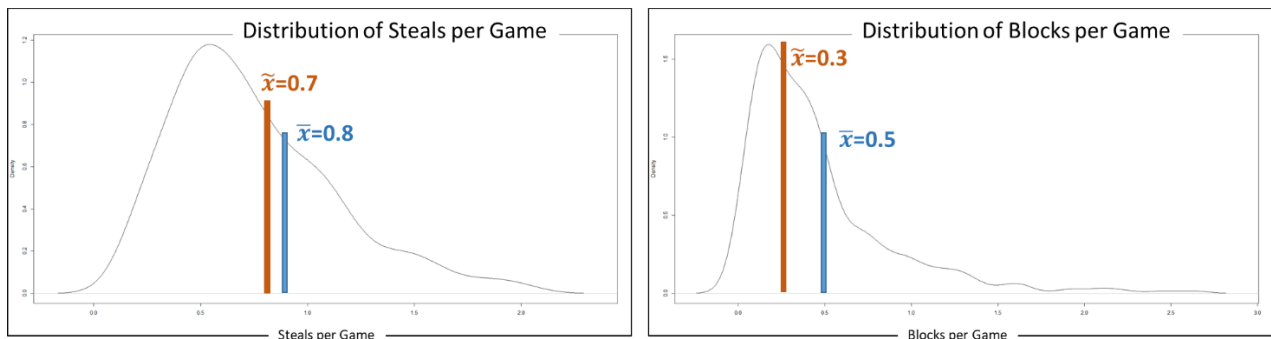


Figure 4 Density Curves of Steals and Blocks per Game.

Utilizing the normalization of the aggregated sums of each of the scoring criteria’s Pearson’s Coefficient of Skewness (#2) it is determined that: FG%, FT%, AST/G, and BLK/G are the scarcest attributes. In contrast, the least scarce statistic is 3PM/G with the remaining attributes being fairly common. Sensitivity analysis in the Discussion Section will further explore this weighting.

FG%	3PM/G	FT%	REB/G	AST/G	STL/G	BLK/G	TOV/G	PPG
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0.14	0.06	0.15	0.11	0.14	0.08	0.13	0.10	0.09
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Figure 5 Criteria Weighting.

Following the weighting criteria the matrix can now be normalized. As described in the methodology, this is done by transforming each of the attributes into non-dimensional attributes, having a value between 0 to 1, allowing for direct comparison.²

	FG%	3PM/G	FT%	REB/G	AST/G	STL/G	BLK/G	TOV/G	PPG
Russell Westbrook	0.111804	0.106069	0.211781	0.122923	0.196408	0.108889	0.032549	0.003746	0.161151
James Harden	0.09753	0.138951	0.223165	0.091918	0.212367	0.09898	0.039899	0	0.147089
Isaiah Thomas	0.118317	0.138483	0.197091	0.025294	0.110057	0.057189	0.014548	0.034609	0.146231
Anthony Davis	0.159918	0.022911	0.160037	0.136644	0.036713	0.081433	0.189371	0.038691	0.140888
DeMar DeRozan	0.132012	0.019157	0.177046	0.056152	0.071936	0.066893	0.014941	0.038469	0.137001
Damian Lillard	0.106076	0.122574	0.162992	0.052359	0.109515	0.056139	0.022679	0.036202	0.13525
DeMarcus Cousins	0.112865	0.07816	0.160975	0.127347	0.085291	0.091324	0.109852	0.023253	0.135168
LeBron James	0.172262	0.071984	0.093943	0.098036	0.164754	0.080697	0.050569	0.019069	0.131972
Kawhi Leonard	0.12268	0.085336	0.156969	0.063436	0.064114	0.121122	0.063211	0.042569	0.126943
Stephen Curry	0.114354	0.176183	0.102328	0.04699	0.124299	0.121135	0.018301	0.031549	0.125761
Kyrie Irving	0.128697	0.105605	0.103048	0.031384	0.108336	0.074096	0.028349	0.03768	0.125301
Karl-Anthony Towns	0.166677	0.052912	0.102493	0.142694	0.048089	0.039814	0.106827	0.036684	0.124804
Kevin Durant	0.147671	0.081066	0.133294	0.093614	0.089682	0.067656	0.135801	0.04088	0.124502

Figure 6 Normalized Matrix.

The normalized matrix is rearranged by ascending PPG to highlight the idiosyncrasies of the data. As an example, the player James Harden (second from the top) has a value of 0 for TOV/G as a result of having highest number, or worst, turnovers in the league. This demonstrates Harden's built-in penalty of this attribute when the weighting criteria is applied and the separation from the positive and negative ideal solution is calculated. The maximum and minimum of each normalized attributes is listed in the top two rows in the figure below with every minimum set to zero. The separation from the positive and negative ideal solutions are the one-dimensional Euclidean distance of each weighted normalized value.

Max	0.026502	0.026502	0.006776	0.006776	0.034333	0.034333	0.00636	0.00636	0.032672	0.032672	0.021205	0.021205	0.034568	0.034568	0.005033	0.005033	0.012396	0.012396
Min	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	FG% D+	FG% D-	3PM D+	3PM D-	FT% D+	FT% D-	REB D+	REB D-	AST D+	AST D-	STL D+	STL D-	BLK D+	BLK D-	TOV D+	TOV D-	PPG D+	PPG D-
Russell Westbrook	0.009301	0.017201	0.002697	0.00408	0.001751	0.032582	0.001632	0.004728	0.002455	0.030217	0.004453	0.016752	0.029561	0.005008	0.004745	0.000288	0	0.012396
James Harden	0.011497	0.015005	0.001432	0.005344	0	0.034333	0.002824	0.003535	0	0.032672	0.005977	0.015228	0.02843	0.006138	0.005033	0	0.001082	0.011315
Isaiah Thomas	0.008299	0.018203	0.00145	0.005326	0.004011	0.030322	0.005387	0.000973	0.01574	0.016932	0.012407	0.008798	0.03233	0.002238	0.002371	0.002662	0.001148	0.011249
Anthony Davis	0.001899	0.024603	0.005895	0.000881	0.009712	0.024621	0.001104	0.005256	0.027024	0.005648	0.008677	0.012528	0.005434	0.029134	0.002057	0.002976	0.001559	0.010838
DeMar DeRozan	0.006192	0.020309	0.006039	0.000737	0.007095	0.027238	0.0042	0.00216	0.021605	0.011067	0.010914	0.010291	0.032269	0.002299	0.002074	0.002959	0.001858	0.010539
Damian Lillard	0.010182	0.016319	0.002062	0.004714	0.009258	0.025076	0.004346	0.002014	0.015823	0.016848	0.012568	0.008637	0.031079	0.003489	0.002248	0.002785	0.001992	0.010404
DeMarcus Cousins	0.009138	0.017364	0.00377	0.003006	0.009568	0.024765	0.001462	0.004898	0.01955	0.013122	0.007155	0.01405	0.017668	0.0169	0.003244	0.001789	0.001999	0.010398
LeBron James	0	0.026502	0.004008	0.002769	0.01988	0.014453	0.002589	0.003771	0.007325	0.025347	0.00879	0.012415	0.026788	0.00778	0.003566	0.001467	0.002245	0.010152
Kawhi Leonard	0.007628	0.018874	0.003494	0.003282	0.010184	0.024149	0.00392	0.00244	0.022808	0.009864	0.002571	0.018634	0.024843	0.009725	0.001758	0.003275	0.002631	0.009765
Stephen Curry	0.008909	0.017593	0	0.006776	0.01859	0.015743	0.004552	0.001807	0.013549	0.019123	0.002569	0.018636	0.031752	0.002816	0.002606	0.002427	0.002722	0.009674
Kyrie Irving	0.006702	0.0198	0.002715	0.004062	0.01848	0.015854	0.005153	0.001207	0.016005	0.016667	0.009806	0.011399	0.030207	0.004361	0.002135	0.002898	0.002758	0.009639
Karl-Anthony Towns	0.000859	0.025643	0.004741	0.002035	0.018565	0.015768	0.000871	0.005488	0.025274	0.007398	0.01508	0.006125	0.018133	0.016435	0.002211	0.002822	0.002796	0.0096
Kevin Durant	0.003783	0.022719	0.003658	0.003118	0.013826	0.020507	0.002759	0.003601	0.018875	0.013797	0.010796	0.010409	0.013676	0.020892	0.001888	0.003145	0.002819	0.009577

Figure 7 Distance from Positive and Negative Ideal Calculated.

Examining the distances from the positive and negative ideal solutions, we observe several "0's" in Figure 7 signifying players closest or furthest away from the given attribute's ideal solution. James Harden is revealed as both the ideal positive solution for FT% and AST/G and the negative ideal solution for TOV/G. With the upper and lower bounds calculated, Equation #7 determines the relative closeness of each player to each attribute. This is indicated in the figure below.

² Up to this point other MCDM techniques follow the same methodology, as this creates a natural ranking.

Max	1	1	1	1	1	1	1	1	1	5.860458
Min	0	0	0	0	0	0	0	0	0	1.21104
	FG%	3PM/G	FT%	REB/G	AST/G	STL/G	BLK/G	TOV/G	PPG	TOTAL
Russell Westbrook	0.649035	0.602042	0.948988	0.743414	0.924852	0.790009	0.14486	0.057258	1	5.860458
James Harden	0.566171	0.788676	1	0.555903	1	0.71812	0.17757	0	0.912743	5.719182
Isaiah Thomas	0.686846	0.786022	0.883161	0.152973	0.518237	0.414914	0.064744	0.528945	0.907417	4.943259
Anthony Davis	0.928344	0.130041	0.717122	0.826394	0.172876	0.590811	0.842804	0.591347	0.674363	5.674003
DeMar DeRozan	0.766343	0.108734	0.79334	0.339593	0.338732	0.48532	0.066494	0.58794	0.850143	4.336639
Damian Lillard	0.615785	0.69572	0.730362	0.316659	0.515687	0.407299	0.100935	0.553293	0.839274	4.775013
DeMarcus Cousins	0.655194	0.44363	0.721327	0.770168	0.401618	0.662569	0.488902	0.355388	0.838769	5.337565
LeBron James	1	0.408575	0.420958	0.592899	0.775798	0.58547	0.225057	0.291441	0.818936	5.119134
Kawhi Leonard	0.712174	0.484359	0.703377	0.383646	0.301901	0.878766	0.281321	0.650615	0.78773	5.183889
Stephen Curry	0.663839	1	0.458529	0.284185	0.585301	0.878857	0.08145	0.482181	0.780392	5.214733
Kyrie Irving	0.747102	0.599408	0.461757	0.189801	0.510135	0.537581	0.126168	0.575888	0.777538	4.525377
Karl-Anthony Towns	0.967577	0.300324	0.459269	0.86298	0.226441	0.288859	0.475439	0.56066	0.774456	4.916005
Kevin Durant	0.857247	0.460125	0.597287	0.566157	0.422296	0.490858	0.604386	0.624799	0.772584	5.395739

Figure 8 Closeness to Overall Ideal Solution.

The proximity to the overall ideal solution is shown in the matrix above. There are several “1’s” in the matrix that indicate, with regards to that attribute, the player is the ideal solution. The final column in the matrix, highlighted in yellow, is the player’s sum score of the closeness for each attribute. Finally, the score for each player is arranged in descending rank order. The top-10 players of this analysis is shown below and will be expanded upon in the following section.

Rank	Player	TOPSIS Score
1	Russell Westbrook	5.860
2	James Harden	5.719
3	Anthony Davis	5.674
4	Kevin Durant	5.396
5	DeMarcus Cousins	5.338
6	Giannis Antetokounmpo	5.271
7	Stephen Curry	5.215
8	Kawhi Leonard	5.184
9	LeBron James	5.119
10	Jimmy Butler	5.034

Figure 9 Final TOPSIS Ranking of Players (Top-10).

IV. Results and Findings

The generated TOPSIS rankings provide tangible results ready to test against the FantasyPros.com pre-season rankings. Serving as the bellwether, the 2017-2018 ESPN end-of-season (www.espn.com/fantasy/) fantasy rankings will be the ranking standard for the pairwise comparison. The ESPN rankings are an appropriate choice as their fantasy basketball data is widely available and their ranking methodology is well known (Stat Dance 2014). Quantitatively, the ESPN fantasy rankings utilize a modified z-score to rank their players which is also the basis for the fantasy experts’ rankings. We can, therefore, posit a conservative analytical bias as the FantasyPros.com rankings should hold an initial advantage over the TOPSIS rankings.

It is prudent to first visually compare the two rankings against the ESPN rankings to garner perspective of the holistic similarities and differences. Orienting to the figure below, the x-axis indicates, from left to right, players with the best (lowest) ESPN rankings to the worst. Whereas the y-axis indicates that respective player’s TOPSIS and expert rankings. The AB line represents the descending ESPN score, while the other two lines show the difference between the projected and final ranking. The closer the TOPSIS and expert rankings are to the AB line the better the

projection. The extent and magnitude of difference cannot be determined visually. Quantifiable analytical techniques will establish a proper comparison.

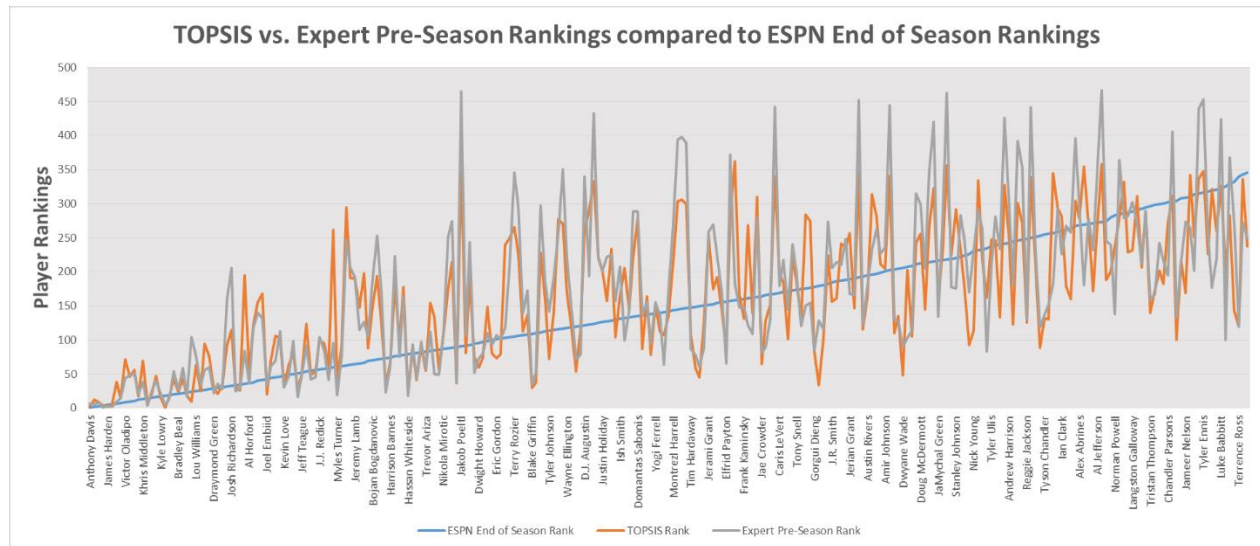


Figure 10 Difference between TOPSIS and Z-score Ranking.

Initial descriptive statistics are calculated from the difference in absolute values between the rankings and the ESPN rankings. This quantifies the magnitude of the individual rankings from the standard. The differences are both normally distributed providing a positive underpinning for continued analysis. The TOPSIS ranking has an average difference of 63 compared to the experts' difference of 70. This lack of accuracy in both rankings is amplified by their large standard deviations (54; 67). Notwithstanding, it speculatively appears that TOPSIS is a slightly better alternative than the experts' rankings. A graphic demonstration of the difference, as density curves, is shown in the figure below.

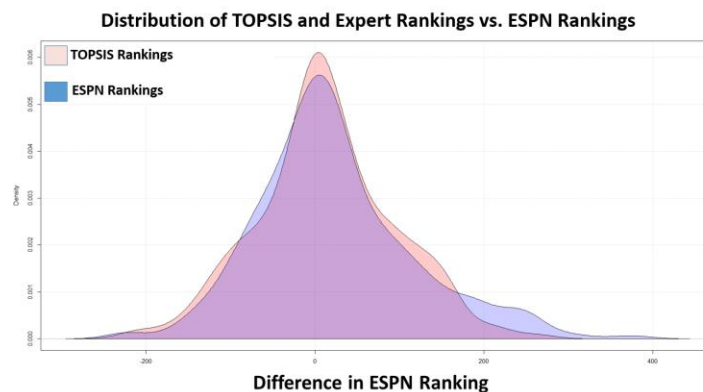


Figure 11 Density curves of the differences between the two rankings.

The figure above does not visually show TOPSIS having a slightly closer proximity to 0, which is preferred. Consequently, a two-sample t-test is conducted in order to test if the unpaired population means are equivalent. The alternate hypothesis, then, is that the two are different and not due to chance. Given that the two populations have unequal variance it is

determined that there is a marginal difference between the two populations (p-value is 0.09) based on $\alpha=0.10$. Given this finding, TOPSIS is marginally more accurate than the experts' rankings and therefore a better ranking method.

In spite of these positive results, the case for using TOPSIS over the experts' rankings needs to be further solidified. Therefore, to supplement the scope of the original hypothesis complementary analysis of the source of uncertainty in fantasy basketball should be conducted. It is not presumptuous to assume that experts can easily project how a good player, such as James Harden, will perform during a season. The abundant qualitative analysis of high visibility players may lend merit to subjective expert judgement. Opportunity may exist in projecting ranks of less established players. This analysis is not trivial as players falling outside of the Top-100 normally occupy half of a fantasy team's roster and are therefore significant contributors. One associated difficulty in these projections is that the players' contributions are highly variable and heavily dependent on playing time.

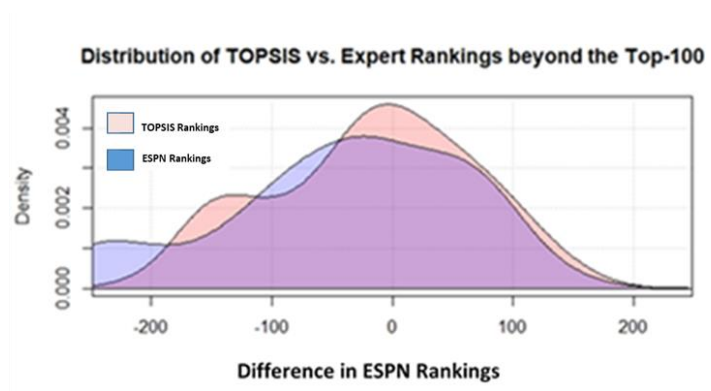


Figure 12 Density curves of difference between the two rankings outside the Top-100.

A visual analysis is shown in the figure above of players ranked outside of ESPN Top-100 yielding similar results in favor of the TOPSIS rankings. The difference in projected versus final rankings is more precise at an average of only 17, whereas FantasyPros.com was considerably higher at an average of 37 spots. A two-sample t-test further indicates a difference exists between the ranking means with a p-value of 0.06. It is conjectured that with less analysis conducted by the experts on the ordinal ranking of players outside of the Top-100, TOPSIS can quantify the interval ranking and thus be more accurate.

IV. Analysis and Discussion

TOPSIS offers users the adeptness to apply a weighted ranking criteria, one of its most robust features for tailored fantasy rankings. Fine tuning ranking criteria in order to distinguish good players from exceptional players requires a sound analytical method to reduce subjectivity.

Whether weighting the statistical categories by scarcity, or normalizing each category and weighting based on magnitude of outliers, TOPSIS allows managers to find well-rounded players that contribute to all categories. Not all basketball positions, however, provide or are valued for well-rounded performance. The Center (C), for example, is known to provide statistics that are not balanced. Specifically, Centers produce a large number of rebounds, blocks, and high FG% while neither accumulating many assists, or 3PM, nor shooting high FT%. The calculated weighting criteria for this analysis values blocks, FG%, FT%, and assists highest therefore the Center position should perform above average on two of the four highest rated categories and below average on the other two highest rated categories. For a typical Center, the positives will nullify the negatives. Thus, for a Center to be ranked higher in the overall rankings they would have to break this prototypical mold and perform a minimum of average on FT% and assists while contributing to the other categories. Two such players: Anthony Davis and DeMarcus Cousins are ranked #3 and #5 overall as their statistics show they are great across the spectrum of all applicable statistics with minimal exception (TOV/G).

TOPSIS Rank	Player	Pos	MPG	FG%	3PM/G	FT%	REB/G	AST/G	STL/G	BLK/G	TO/G	PPG
	AVG			46%	0.9	76%	4.2	2.2	0.7	0.2	1.3	10.3
	Weight			0.14	0.06	0.15	0.11	0.14	0.08	0.13	0.10	0.09
3	Anthony Davis	C	36.1	51%	0.5	80%	11.8	2.1	1.3	2.2	2.4	28.0
5	DeMarcus Cousins	C	34.2	45%	1.8	77%	11.0	4.6	1.4	1.3	3.7	27.0

Figure 13 Top-10 TOPSIS Ranked Centers.

TOPSIS strongly penalizes players who do not contribute to scarce categories. Hence it is appropriate to conduct sensitivity on the criteria weighting thus TOPSIS is re-applied with all criteria set to equal weight producing descriptive statistics with rankings nearly equivalent to the original rankings with a mean difference of 64 (54σ) and p-value of 0.002. A t-test reveals the two sample means are not from the same population. This difference can be visually discerned in the figure below. The modified TOPSIS rankings seem to retain the preciseness of the weighted rankings, but the leftward shift of the mean away from zero indicates they are not as accurate. Therefore, the original weighting makes the desired adjustment for scarcer attributes and properly values players that over-perform in those categories. Without weighting statistical categories the TOPSIS method achieves little more than perform to an overly-complicated z-score calculation.

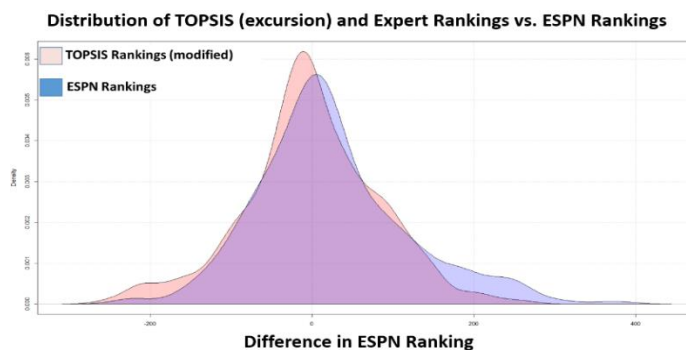


Figure 14 Sensitivity Analysis Excursion on TOPSIS Rankings.

Balanced rosters are essential to fantasy basketball teams as standard fantasy settings require five set position spots for the Point Guard (PG), Shooting Guard (SG), Small Forward (SF), Power Forward (PF), and Center (C) on a team. The remaining roster spots are usually reserved for utility players that can fill any position. In the talent pool, SG and C account for half the players while PF accounts for the least at around 12%. With this in mind it is worth exploring how each TOPSIS projected position deviated from the ESPN rankings.

Position	Average Diff.	SD of Diff
PG	52.2	50.7
SF	62.2	52.4
SG	63.4	55.2
PF	66.6	51.9
C	69.0	55.0
Total	62.7	53.7

Figure 15 Differences in Rankings by Player Position

The figure above indicates that TOPSIS is more accurate in projecting Point Guards' rankings than Centers'. Centers show both a larger absolute difference and more variability in the rankings, as indicated by the standard deviation. Surveying the Center position shows of the 60 Centers, six players' spread between their TOPSIS and ESPN rankings are beyond the statistical threshold of a random occurrence and are hence outliers. For these players it appears the 2016-2017 NBA season statistics did not project well for their next year's performance principally due to the minutes per game which did not represent their intrinsic capabilities. This provides credence to the general axiom that the more/less minutes per game a player plays the more/less opportunity they will have to accumulate individual statistics. The axiom is supported through calculation of the Pearson correlation coefficient (R), which has an incredibly strong negative relationship ($R = -0.93$) between minutes played and player ranking. Thus it is reasonable to conclude that since fantasy experts understand the impact of minutes per game, and they make their rankings prior to the start of the season, their expert judgement should improve accuracy over the rankings which are calculated at the end of season. In contrast, TOPSIS shows that it adds more value as a ranking methodology due to its consistency in finding desired players attributes, especially outside the Top-100.

V. Conclusion

As the appetite for more basketball analytics grows, innovative techniques continue to develop. The game has already become easier to model through the proliferation of new statistics captured through the integration of wearable technology that tracks a player's movement on the court (Frye 2018). By tracking a player's movement on the court an analyst can determine if the player is hustling by how fast and long he sprints while playing defense. This "hustle statistic" (Concepcion 2018) is now being tracked by the NBA and is a representation of a players' effort on the court, hypothesized to relate to their ability to create statistical opportunities. Other statistics such as number of times a player gets the ball in the paint area, otherwise known as paint touches (www.nba.com), is correlated to scoring and higher FG%, as

these shots are closer to the rim and have a higher probability of being made. As more advanced collection techniques, and subsequent statistics develop, TOPSIS provides analysts and fantasy managers an adept tool to incorporate these evolutions in their attribute evaluation.

This paper demonstrates the application of TOPSIS to rank fantasy basketball players providing notable improvement over the experts' post-analysis subjective judgements at FantasyPros.com. The TOPSIS rankings performed admirably against the expert rankings principally due to its adroit accuracy and precision in parallel to the ESPN end-of-season rankings. This is principally because fantasy experts use a host of techniques to create their rankings, blending quantitative and qualitative factors in order to project bottom-up rankings. This mixing of subjective and objective criterion does not show clear methodological reasoning or produce superior results. The approach is mired by weighting all criterion equally, ignoring the insight that some statistics are easier to achieve than others. Conversely, TOPSIS systematically ranks by predetermined preferences. These preferences are calculated by weighting criteria based on the scarcity of the attribute. The consolidated set of metrics, normalization, and weighting objectively ranks players based on their distances from the positive and negative ideal solutions. The best choices have the shortest distance to the most positive ideal solution, an approach validated by this investigation. The application of TOPSIS provides an improved entry level technique to create objective fantasy basketball rankings.

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